Handling Infinite Branching WSTS

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- Moreover, multiple decidability results are known on WSTS.
- However, most results and techniques known suppose finite branching.
- Developing from a theory elaborated by Finkel and Goubault-Larrecq, we introduce a way to work with infinitely branching WSTS.

Ordered transition systems

$S = (X, \rightarrow_S, \leq)$ where

- X set,
- $\rightarrow_S \subseteq X \times X$,
- $\leq$ quasi-ordering X.

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$S = (X, \rightarrow_S, \leq)$ where

- X set: recursively enumerable,
- $\rightarrow_S \subseteq X \times X$: decidable,
- $\leq$ quasi-ordering X: decidable.

Well-ordered transition system (WSTS)

A *WSTS* is an ordered transition system $(X, \rightarrow, \leq)$ with

- well-quasi-ordering: $\forall x_0, x_1, \ldots \ \exists i < j \ \text{ s.t. } \ x_i \leq x_j$,
- monotony:

$$\begin{array}{cccc} \forall & x & \rightarrow & y \\ & \wedge\!\backslash & & \wedge\!\backslash \\ & x' & \xrightarrow{*} & y' \quad \exists \end{array}$$

(Some) types of monotony

Standard monotony:

$$\begin{array}{ccccc} \forall & x & \rightarrow & y & \\ & \rotatebox{90}{$\leq$} & & \rotatebox{90}{$\leq$} & \\ & x' & \xrightarrow{*} & y' & \exists \end{array}$$

(Some) types of monotony

Strong monotony:

$$\begin{array}{cccc} \forall & x & \rightarrow & y \\ & \wedge\!\!\backslash & & \wedge\!\!\backslash \\ & x' & \rightarrow & y' \quad \exists \end{array}$$

(Some) types of monotony

Transitive monotony:

$$\begin{array}{llll} \forall & x & \rightarrow & y \\ & \leqslant & & \leqslant \\ & x' & \xrightarrow{+} & y' \quad \exists \end{array}$$

(Some) types of monotony

Strict monotony:

$$\begin{array}{cccc} \forall\ x & \rightarrow & y & \\ \wedge & & \wedge & \\ x' & \xrightarrow{*} & y' & \exists \end{array}$$

Branching

A WSTS $(X, \rightarrow, \leq)$ is finitely branching if Post(x) is finite for every $x \in X$.

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- Parameterized WSTS,

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- ω-Petri nets (Geeraerts, Heussner, Praveen & Raskin 2013),
- Parameterized WSTS,
- etc.

Effectiveness

A WSTS $(X, \to, \leq)$ is post-effective if it is possible to compute $|\text{Post}(x)|$ for every $x \in X$.

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A WSTS $(X, \rightarrow, \leq)$ is post-effective if it is possible to compute $|\text{Post}(x)|$ for every $x \in X$.

Remark

If $\text{Post}(x)$ is finite, then it is computable by minimal hypotheses. Therefore, our definition generalizes post-effectiveness for finitely branching WSTS.

Termination

Input: $(X, \rightarrow, \leq)$ a WSTS, $x_0 \in X$.

Question: $\exists x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow \ldots ?$

Theorem (Finkel & Schnoebelen 2001)

Decidable for finitely branching post-effective WSTS with transitive monotony.

Termination

Input: $(X, \rightarrow, \leq)$ a WSTS, $x_0 \in X$.

Question: $\exists x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow \ldots$?

Theorem (Blondin, Finkel & McKenzie in progress)

Undecidable for infinitely branching post-effective WSTS with transitive monotony.

Boundedness

Input: $(X, \rightarrow, \leq)$ a WSTS, $x_0 \in X$.

Question: $\mathrm{Post}^*(x_0)$ finite?

Theorem (Finkel & Schnoebelen 2001)

Decidable for finitely branching post-effective WSTS with strict monotony.

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Coverability

Input: $(X, \to, \leq)$ a WSTS, $x_0, x \in X$.

Question: $x_0 \xrightarrow{*} x' \geq x$?

Theorem (Abdulla, Cerans, Jonsson & Tsay 2000; Finkel & Schnoebelen 2001)

Decidable for some classes of infinitely branching WSTS.

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Decidable for some classes of infinitely branching WSTS.

Control-state maintainability

Input: $(X, \rightarrow, \leq)$ a WSTS, $x_0 \in X$ and $\{t_1, \ldots, t_n\} \subseteq X$.

Question: $\exists$ maximal execution $x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow \ldots$ such that $\forall i\ x_i \in \uparrow\{t_1, \ldots, t_n\}$?

Theorem (Finkel & Schnoebelen 2001)

Decidable for finitely branching post-effective WSTS with stuttering monotony.

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Undecidable for infinitely branching post-effective WSTS with stuttering monotony.

Downward closure

$$\downarrow D = \{x \in X : \exists d \in D \; x \leq d\}.$$

Ideals

$I \subseteq X$ is an *ideal* if it is

- downward closed: $I = \downarrow I$,
- directed: $a, b \in I \implies \exists c \in I$ s.t. $a \leq c$ and $b \leq c$.

Theorem (Finkel & Goubault-Larrecq 2009)

Every downward closed set in X is a finite union of ideals of X.

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Corollary (FGL 2009; Blondin, Finkel & McKenzie in progress)

Every downward closed subset decomposes canonically as the union of its maximal ideals.

Completion (FGL 2009; Blondin, Finkel & McKenzie in progress)

The *completion* of $S = (X, \rightarrow_S, \leq)$ is $\widehat{S} = (\widehat{X}, \rightarrow_{\widehat{S}}, \subseteq)$ such that

- $\widehat{X} = \mathrm{Ideals}(X)$,
- $I \rightarrow_{\widehat{S}} J$ if J appears in the canonical decomposition of $\downarrow \mathrm{Post}(I)$.

Theorem (FGL 2009; Blondin, Finkel & McKenzie in progress)

Let $S = (X, \rightarrow_S, \leq)$ be a WSTS, then

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- $\widehat{S}$ is finitely branching.
- $\widehat{S}$ has strong monotony.

Theorem (FGL 2009; Blondin, Finkel & McKenzie in progress)

Let $S = (X, \rightarrow_S, \leq)$ be a WSTS, then

- $\widehat{S}$ is finitely branching.
- $\widehat{S}$ has strong monotony.
- $\widehat{S}$ is a WSTS iff S is a ω^2-WSTS iff $A \leq^{\#} B \Leftrightarrow \uparrow A \subseteq \uparrow B$ is a wqo (by Jančar 1999).

Ideals in $\mathbb{N}^d$

$I \subseteq \mathbb{N}^d$ is an ideal iff $I = \downarrow x_1 \times \cdots \times \downarrow x_d$ with $x_i \in \mathbb{N}$ or $x_i = \mathbb{N}$.

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Representation

- $\downarrow 5 \times \mathbb{N} \times \downarrow 10$ can be represented by $(5, \omega, 10)$,

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Representation

- $\downarrow 5 \times \mathbb{N} \times \downarrow 10$ can be represented by $(5, \omega, 10)$,
- $\downarrow 5 \times \mathbb{N} \times \downarrow 10 \subseteq \mathbb{N} \times \mathbb{N} \times \downarrow 20$ can be tested by $(5, \omega, 10) \leq (\omega, \omega, 20)$.

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Example

VAS $A = \{(2, -3, -5), (4, 5, -1), (-6, -2, 5)\}$ and ideal $I = \downarrow 5 \times \mathbb{N} \times \downarrow 10$:

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Example

VAS $A = \{(2,-3,-5),(4,5,-1),(-6,-2,5)\}$ and ideal $I = {\downarrow}5 \times \mathbb{N} \times {\downarrow}10$:

$$(5,\omega,10) + (2,-3,-5) = (7,\omega,5)$$

$${\downarrow}\mathrm{Post}(I) = {\downarrow}7 \times \mathbb{N} \times {\downarrow}5$$

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$$(5,\omega,10) + (4,5,-1) = (9,\omega,9)$$

$$\downarrow \text{Post}(I) = \downarrow 7 \times \mathbb{N} \times \downarrow 5 \cup \downarrow 9 \times \mathbb{N} \times \downarrow 9$$

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Example

VAS $A = \{(2,-3,-5),(4,5,-1),(-6,-2,5)\}$ and ideal $I = \downarrow 5 \times \mathbb{N} \times \downarrow 10$:

$$(5,\omega,10) + (-6,-2,5) = \emptyset$$

$$\downarrow \mathrm{Post}(I) = \downarrow 7 \times \mathbb{N} \times \downarrow 5 \cup \downarrow 9 \times \mathbb{N} \times \downarrow 9$$

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Example

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Example

VAS $A = \{(2, -3, -5), (4, 5, -1), (-6, -2, 5)\}$ and ideal $I = \downarrow 5 \times \mathbb{N} \times \downarrow 10$:

$$\mathrm{Post}_{\widehat{S}}(I) = \{\downarrow 9 \times \mathbb{N} \times \downarrow 9\}$$

Coverability

Input: $(X, \rightarrow, \leq)$ a WSTS, $x_0, x \in X$.

Question: $x_0 \xrightarrow{*} x' \geq x$?

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Question: $x_0 \in \uparrow \mathrm{Pre}^*(\uparrow x)$?

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Question: $x_0 \in \uparrow \mathrm{Pre}^*(\uparrow x)$?

Backward method (Abdulla, Cerans, Jonsson & Tsay 2000)

Compute sequence converging to $\uparrow \mathrm{Pre}^*(\uparrow x)$:

$$\begin{array}{rcll} Y_0 & = & \uparrow x & \\ Y_1 & = & Y_0 & \cup \ \uparrow \mathrm{Pre}(Y_0) \\ \vdots & & \vdots & \qquad \vdots \\ Y_n & = & Y_{n-1} & \cup \ \uparrow \mathrm{Pre}(Y_{n-1}) \end{array}$$

and verify if $x_0 \in Y_n$.

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and verify if $x_0 \in Y_n$. Computing Pre not always effecient!

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Question: $x \in \downarrow \mathrm{Post}^*(x_0)$?

Theorem (Blondin, Finkel & McKenzie in progress)

Coverability is decidable for WSTS with post-effective completion.

Proof: two semi-algorithms to decide coverability

Coverability:

- Enumerate execution $\downarrow x_0 \xrightarrow{*}_{\widehat{S}} I$,
- Accept if $x \in I$.

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Non coverability:

- Enumerate $D \subseteq X$ downward closed

Proof: two semi-algorithms to decide coverability

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- Enumerate execution $\downarrow x_0 \xrightarrow{*}_{\widehat{S}} I$,
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Non coverability:

- Enumerate $D = I_1 \cup \ldots \cup I_k$

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- Enumerate $D \subseteq X$ downward closed, $x_0 \in D$

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- Enumerate $D \subseteq X$ downward closed, $\downarrow x_0 \subseteq D$

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- Enumerate execution $\downarrow x_0 \xrightarrow{*}_{\widehat{S}} I$,
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Proof: two semi-algorithms to decide coverability

Coverability:

- Enumerate execution $\downarrow x_0 \xrightarrow{*}_{\widehat{S}} I$,
- Accept if $x \in I$.

Non coverability:

- Enumerate $D \subseteq X$ downward closed, $\exists i$ t.q. $\downarrow x_0 \subseteq I_i$

Proof: two semi-algorithms to decide coverability

Coverability:

- Enumerate execution $\downarrow x_0 \xrightarrow{*}_{\widehat{S}} I$,
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Non coverability:

- Enumerate $D \subseteq X$ downward closed, $x_0 \in D$

Proof: two semi-algorithms to decide coverability

Coverability:

- Enumerate execution $\downarrow x_0 \xrightarrow{*}_{\widehat{S}} I$,
- Accept if $x \in I$.

Non coverability:

- Enumerate $D \subseteq X$ downward closed, $x_0 \in D$ and $\downarrow \mathrm{Post}_S(D) \subseteq D$

Proof: two semi-algorithms to decide coverability

Coverability:

- Enumerate execution $\downarrow x_0 \xrightarrow{*}_{\widehat{S}} I$,
- Accept if $x \in I$.

Non coverability:

- Enumerate $D \subseteq X$ downward closed, $x_0 \in D$ and $\downarrow \mathrm{Post}_S(I_1 \cup \ldots \cup I_k) \subseteq I_1 \cup \ldots \cup I_k$

Proof: two semi-algorithms to decide coverability

Coverability:

- Enumerate execution $\downarrow x_0 \xrightarrow{*}_{\widehat{S}} I$,
- Accept if $x \in I$.

Non coverability:

- Enumerate $D \subseteq X$ downward closed, $x_0 \in D$ and $\downarrow \mathrm{Post}_S(I_1) \cup \ldots \cup \downarrow \mathrm{Post}_S(I_k) \subseteq I_1 \cup \ldots \cup I_k$

Proof: two semi-algorithms to decide coverability

Coverability:

- Enumerate execution $\downarrow x_0 \xrightarrow{*}_{\widehat{S}} I$,
- Accept if $x \in I$.

Non coverability:

- Enumerate $D \subseteq X$ downward closed, $x_0 \in D$ and

$$\underbrace{(J_{1,1} \cup \ldots \cup J_{1,n_1})}_{Post_{\widehat{S}}(I_1)=\{J_{1,1},\ldots,J_{1,n_1}\}} \cup \ldots \cup \underbrace{(J_{k,1} \cup \ldots \cup J_{k,n_k})}_{Post_{\widehat{S}}(I_k)=\{J_{k,1},\ldots,J_{k,n_k}\}} \subseteq I_1 \cup \ldots \cup I_k$$

Proof: two semi-algorithms to decide coverability

Coverability:

- Enumerate execution $\downarrow x_0 \xrightarrow{*}_{\widehat{S}} I$,
- Accept if $x \in I$.

Non coverability:

- Enumerate $D \subseteq X$ downward closed, $x_0 \in D$ and $\exists i, j, i'$ t.q. $J_{i,j} \subseteq I_{i'}$

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Coverability:

- Enumerate execution $\downarrow x_0 \xrightarrow{*}_{\widehat{S}} I$,
- Accept if $x \in I$.

Non coverability:

- Enumerate $D \subseteq X$ downward closed, $x_0 \in D$ and $\downarrow \mathrm{Post}_S(D) \subseteq D$,

Proof: two semi-algorithms to decide coverability

Coverability:

- Enumerate execution $\downarrow x_0 \xrightarrow{*}_{\widehat{S}} I$,
- Accept if $x \in I$.

Non coverability:

- Enumerate $D \subseteq X$ downward closed, $x_0 \in D$ and $\downarrow \mathrm{Post}_S(D) \subseteq D$,
- Reject if $x \notin D$.

Proof: two semi-algorithms to decide coverability

Coverability:

- Enumerate execution $\downarrow x_0 \xrightarrow{*}_{\widehat{S}} I$,
- Accept if $x \in I$.

Non coverability:

- Enumerate $D \subseteq X$ downward closed, $x_0 \in D$ and $\downarrow \mathrm{Post}_S(D) \subseteq D$,
- Reject if $\downarrow x \not\subseteq I_1 \cup \ldots \cup I_k$.

Proof: two semi-algorithms to decide coverability

Coverability:

- Enumerate execution $\downarrow x_0 \xrightarrow{*}_{\widehat{S}} I$,
- Accept if $x \in I$.

Non coverability:

- Enumerate $D \subseteq X$ downward closed, $x_0 \in D$ and $\downarrow \mathrm{Post}_S(D) \subseteq D$,
- Reject if $\forall i \downarrow x \not\subseteq I_i$.

Proof: two semi-algorithms to decide coverability

Coverability:

- Enumerate execution $\downarrow x_0 \xrightarrow{*}_{\widehat{S}} I$,
- Accept if $x \in I$.

Non coverability:

- Enumerate $D \subseteq X$ downward closed, $x_0 \in D$ and $\downarrow \mathrm{Post}_S(D) \subseteq D$,
- Reject if $x \notin D$. Witness: $D = \downarrow \mathrm{Post}_S^*(x_0)$

Termination

Input: $(X, \rightarrow, \leq)$ a WSTS, $x_0 \in X$.

Question: $\exists x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow \ldots ?$

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Theorem (Blondin, Finkel & McKenzie in progress)

Termination is undecidable, even for post-effective ω^2-WSTS with strong and strict monotony, and with post-effective completion.

Termination

Input: $(X, \to, \leq)$ a WSTS, $x_0 \in X$.

Question: $\exists x_0 \to x_1 \to x_2 \to \ldots$?

Proof

Structural termination is undecidable for Transfer Petri nets (Dufourd, Jančar & Schnoebelen 1999). Structural termination reduces to termination by adding a new element that branches on every other elements.

Execution boundedness

Input: $(X, \rightarrow, \leq)$ a WSTS, $x_0 \in X$.

Question: $\exists k$ bounding length of executions?

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Remark

Termination and execution boundedness are the same in finitely branching WSTS.

Relating executions of S and $\widehat{S}$

Let $S = (X, \rightarrow_S, \leq)$ be a WSTS, then

- if $x \xrightarrow{k}_S y$, then for every ideal $I \supseteq \downarrow x$ there exists an ideal $J \supseteq \downarrow y$ such that $I \xrightarrow{k}_{\widehat{S}} J$,
- if $I \xrightarrow{k}_{\widehat{S}} J$, then for every $y \in J$ there exists $x \in I$ such that $x \xrightarrow{*}_S y' \geq y$.

Relating executions of S and $\widehat{S}$

Let $S = (X, \rightarrow_S, \leq)$ be a WSTS with transitive monotony, then

- if $x \xrightarrow{k}_S y$, then for every ideal $I \supseteq \downarrow x$ there exists an ideal $J \supseteq \downarrow y$ such that $I \xrightarrow{k}_{\widehat{S}} J$,
- if $I \xrightarrow{k}_{\widehat{S}} J$, then for every $y \in J$ there exists $x \in I$ such that $x \xrightarrow{\geq k}_S y' \geq y$.

Relating executions of S and $\widehat{S}$

Let $S = (X, \rightarrow_S, \leq)$ be a WSTS with strong monotony, then

- if $x \xrightarrow{k}_S y$, then for every ideal $I \supseteq \downarrow x$ there exists an ideal $J \supseteq \downarrow y$ such that $I \xrightarrow{k}_{\widehat{S}} J$,
- if $I \xrightarrow{k}_{\widehat{S}} J$, then for every $y \in J$ there exists $x \in I$ such that $x \xrightarrow{k}_S y' \geq y$.

Theorem (Blondin, Finkel & McKenzie in progress)

Execution boundedness is decidable for ω^2-WSTS with transitive monotony, and with post-effective completion.

Theorem (Blondin, Finkel & McKenzie in progress)

Execution boundedness is decidable for ω^2-WSTS with transitive monotony, and with post-effective completion.

Proof

Executions are bounded in S iff bounded in $\widehat{S}$. Since $\widehat{S}$ is finitely branching, it suffices to solve termination in $\widehat{S}$.

Control-state maintainability

Input: $(X, \rightarrow, \leq)$ a WSTS, $x_0 \in X$ and $\{t_1, \ldots, t_n\} \subseteq X$.

Question: $\exists$ maximal execution $x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow \ldots$ such that $\forall i \; x_i \in \uparrow\{t_1, \ldots, t_n\}$?

Control-state maintainability

Input: $(X, \to, \leq)$ a WSTS, $x_0 \in X$ and $\{t_1, \ldots, t_n\} \subseteq X$.

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Theorem (Blondin, Finkel & McKenzie in progress)

Control-state maintainability is undecidable, even for post-effective ω^2-WSTS with strong and strict monotony, and with post-effective completion.

Control-state maintainability boundedness

Input: $(X, \to, \leq)$ a WSTS, $x_0 \in X$ and $\{t_1, \ldots, t_n\} \subseteq X$.

Question: $\exists k$ bounding lengths of executions $x_0 \to x_1 \to x_2 \to \ldots$ such that $\forall i\ x_i \in \uparrow\{t_1, \ldots, t_n\}$?

Control-state maintainability boundedness

Input: $(X, \rightarrow, \leq)$ a WSTS, $x_0 \in X$ and $\{t_1, \ldots, t_n\} \subseteq X$.

Question: $\exists k$ bounding lengths of executions $x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow \ldots$ such that $\forall i \; x_i \in \uparrow \{t_1, \ldots, t_n\}$?

Remark

Control-state maintainability and control-state maintainability boundedness are (almost) the same in finitely branching WSTS.

Theorem (Blondin, Finkel & McKenzie in progress)

Control-state maintainability boundedness is decidable for ω^2-WSTS with transitive monotony, and with post-effective completion.

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Control-state maintainability boundedness is decidable for ω^2-WSTS with transitive monotony, and with post-effective completion.

Proof

"Good" executions are bounded in S iff "good" executions are bounded in $\widehat{S}$. Since $\widehat{S}$ is finitely branching, it suffices to solve control-state maintainability in $\widehat{S}$.

Boundedness

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Question: $\mathrm{Post}^*(x_0)$ finite?

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Question: $\text{Post}^*(x_0)$ finite?

Theorem (Blondin, Finkel & McKenzie in progress)

Boundedness is decidable for post-effective WSTS with strict monotony.

Boundedness

Input: $(X, \rightarrow, \leq)$ a WSTS, $x_0 \in X$.

Question: $\text{Post}^*(x_0)$ finite?

Proof

Build a finite reachability tree as in (Finkel & Schnoebelen 2001) returning "unbounded" if some infinite $\text{Post}(x)$ is encountered.

Open questions

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- Other problems can be solved for infinitely branching WSTS?
- What other applications has the completion?

Thank you! Merci!