# Approaching the Coverability Problem Continuously

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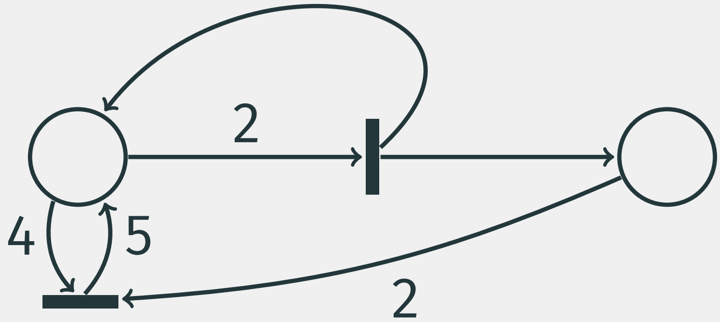
# (Discrete) Petri nets



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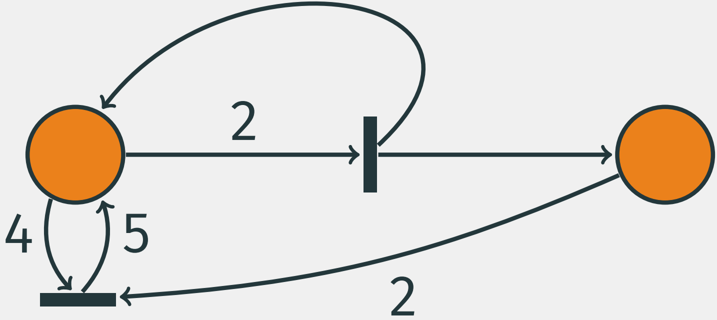
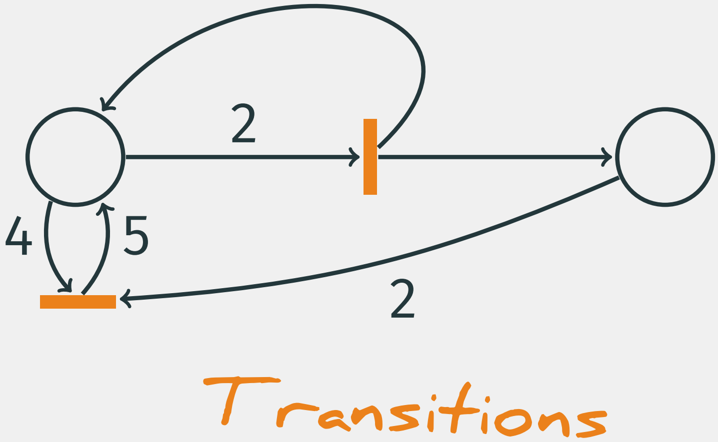



Places

# (Discrete) Petri nets



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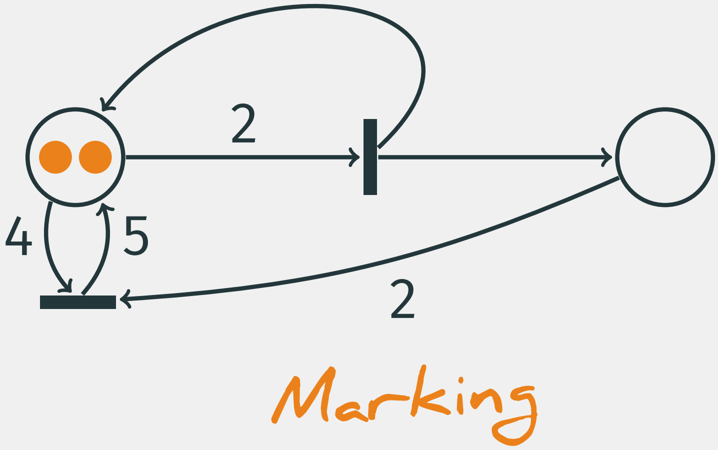

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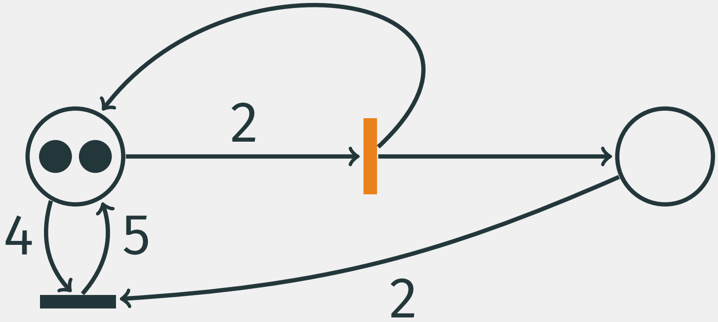

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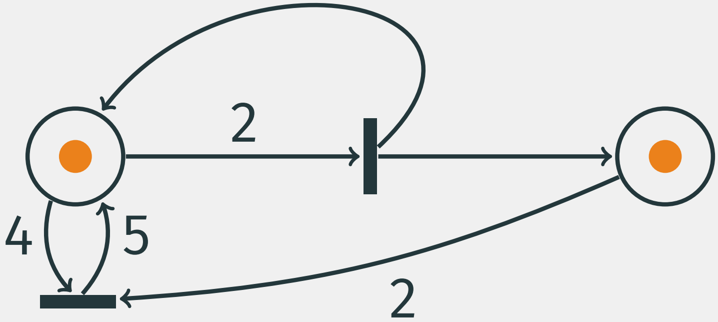

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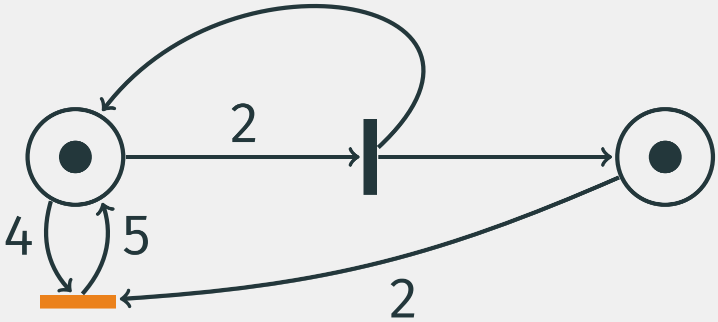

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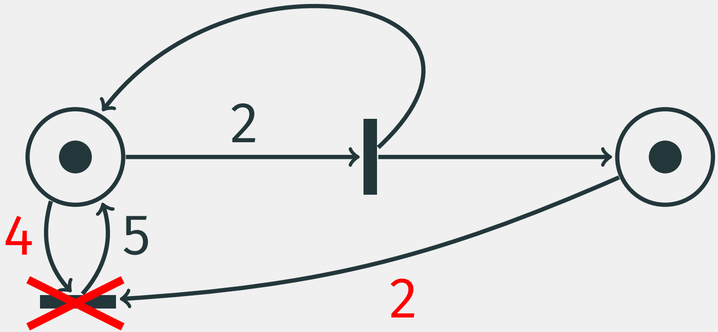

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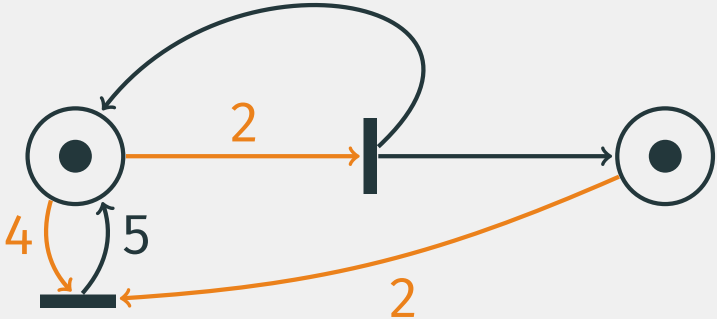
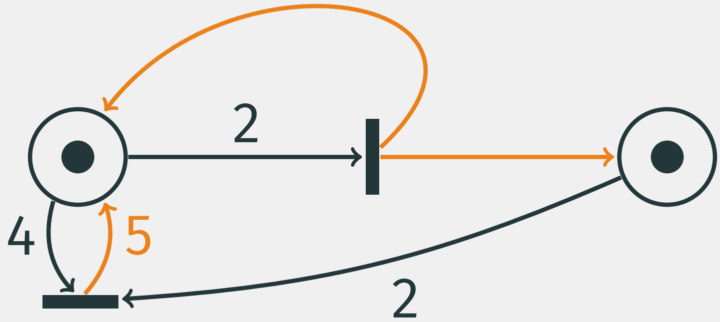



$$Pre = \begin{pmatrix} 4 & 2 \\ 2 & 0 \end{pmatrix}$$

# (Discrete) Petri nets




$$Post = \begin{pmatrix} 5 & 1 \\ 0 & 1 \end{pmatrix}$$

Lamport mutual exclusion "1-bit algorithm"

Process 1

Process 2

Lamport mutual exclusion "1-bit algorithm"

# Verifying safety with Petri nets


Lamport mutual exclusion "1-bit algorithm"

# Verifying safety with Petri nets

```
while True:
  x = True
  while y: pass
  # critical section
  x = False
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while True:
★ y = True
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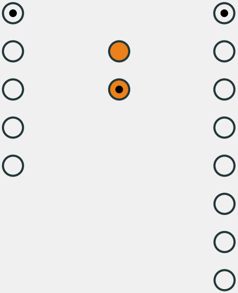

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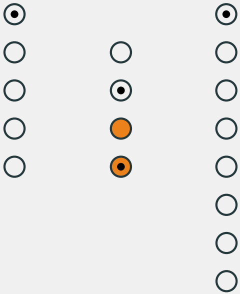

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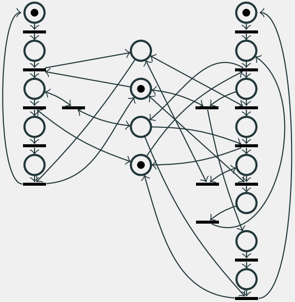

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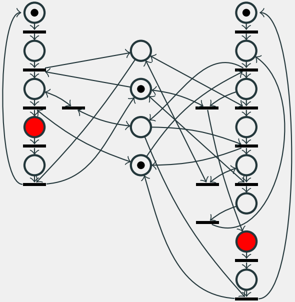

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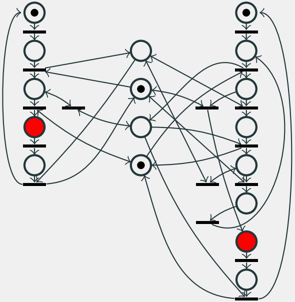

Processes at both critical sections $\iff$ each  $\geq 1$

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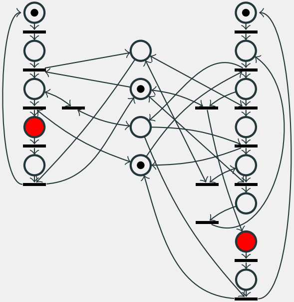
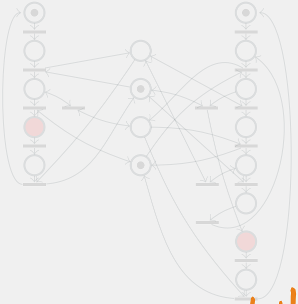


Processes at both critical sections $\iff$ each  $\geq 1$

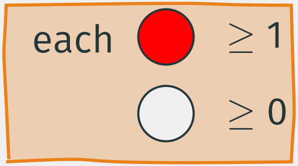
 $\geq 0$

# Verifying safety with Petri nets



Coverability problem

Processes at both critical sections $\iff$



# Coverability problem

**Problem**

| | |
|---|---|
| Input: | Petri net $\mathcal{N}$, initial marking $\boldsymbol{m}_0$, target marking $\boldsymbol{m}$ |
| Question: | Is some $\boldsymbol{m}' \geq \boldsymbol{m}$ reachable from $\boldsymbol{m}_0$ in $\mathcal{N}$? |

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EXPSPACE-complete

Lipton STOC'76, Rackoff TCS'78

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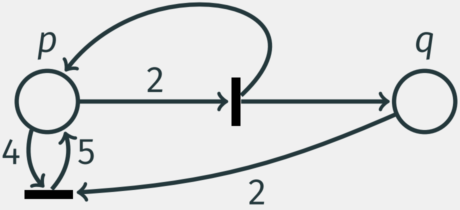
# EXPSPACE-complete

Lipton STOC'76, Rackoff TCS'78

How can we be more efficient?

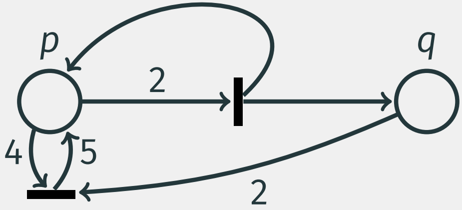
# Over-approximating coverability: state equation




$\boldsymbol{m}_0$ can cover $\boldsymbol{m} \implies$

$$\exists \boldsymbol{v} \geq \mathbf{0} \text{ s.t. } \boldsymbol{m}_0 + (\mathbf{Post} - \mathbf{Pre}) \cdot \boldsymbol{v} \geq \boldsymbol{m}$$

# Over-approximating coverability: state equation




$$\boldsymbol{m}_0 \text{ can cover } \quad \boldsymbol{m} \quad \Longrightarrow$$

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State equation

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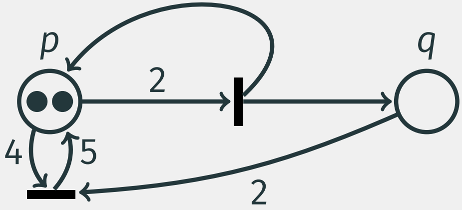
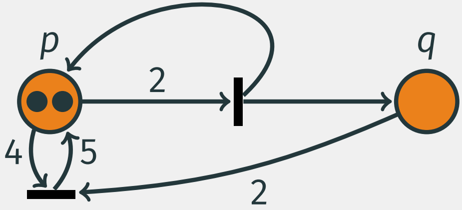



$(2\ \ 0)$ can cover $\boldsymbol{m} \implies$

$$2 + x - y \geq \boldsymbol{m}(p)$$
$$0 - 2x + y \geq \boldsymbol{m}(q)$$

# Over-approximating coverability: state equation

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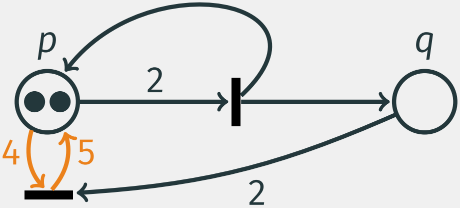


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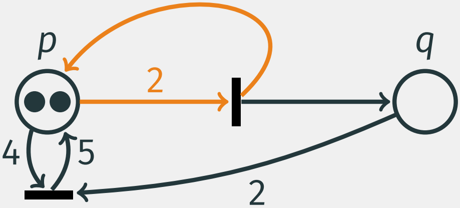


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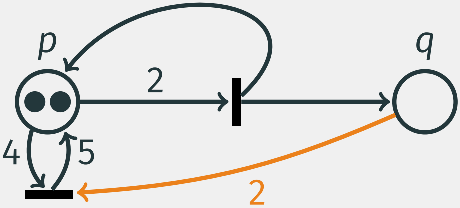


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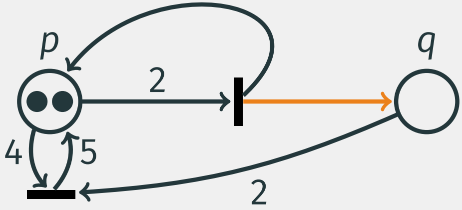


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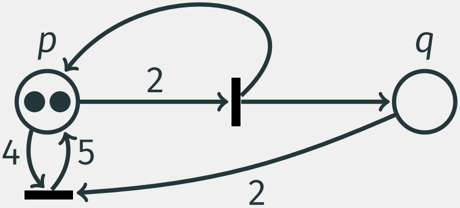
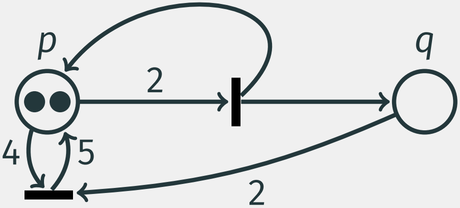



$(2 \;\; 0)$ can cover $(0 \;\; 3) \Longrightarrow$

$$\begin{aligned} 2 + x - y &\geq 0 \\ 0 - 2x + y &\geq 3 \end{aligned}$$

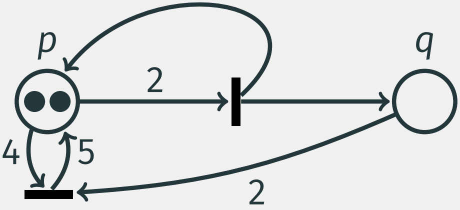
# Over-approximating coverability: state equation




$(2\ \ 0)$ can cover $(0\ \ 3) \Longrightarrow$

$$\begin{aligned} 2 + x - y &\geq 0 \\ 0 - 2x + y &\geq 3 \end{aligned} \quad ✗$$

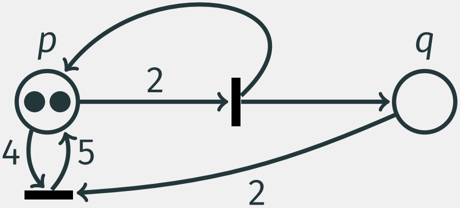
# Over-approximating coverability: state equation




$(2 \;\; 0)$ can cover $(0 \;\; 2) \implies$

$$
\begin{aligned}
2 + x - y &\geq 0 \\
0 - 2x + y &\geq 2
\end{aligned}
$$

# Over-approximating coverability: state equation




$(2 \ \ 0)$ can cover $(0 \ \ 2) \Longrightarrow$

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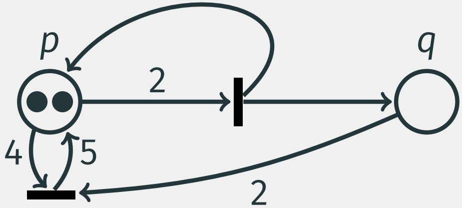


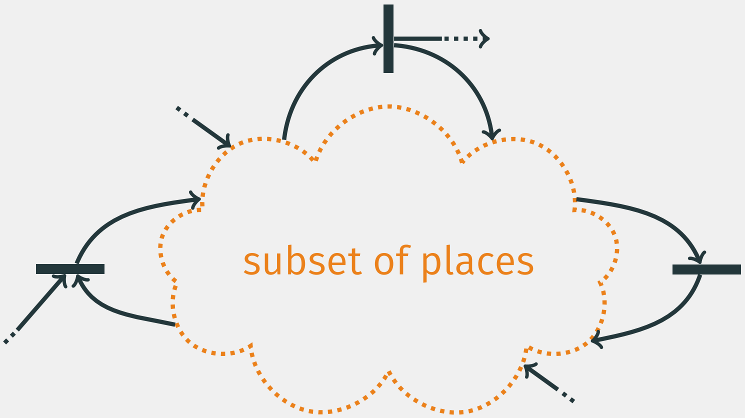

$(2 \;\; 0)$ can cover $(0 \;\; 2) \Longrightarrow$

Yet, not coverable...

$$2 + 0 - 2 \geq 0$$
$$0 - 0 + 2 \geq 2$$

✓

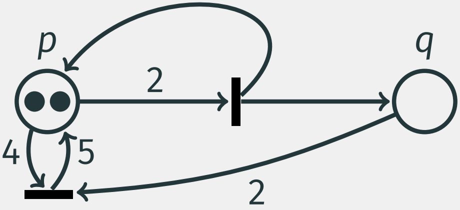
# Traps



# Traps

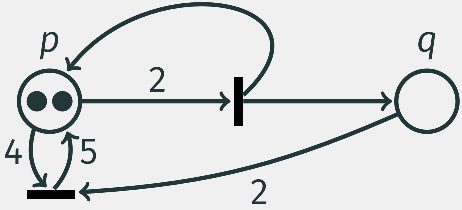
Once trap has tokens, it will always have tokens

# Over-approximating coverability: trap constraints




$(2 \;\; 0)$ can cover $(0 \;\; 2)$?

# Over-approximating coverability: trap constraints

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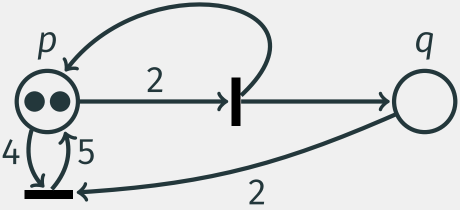
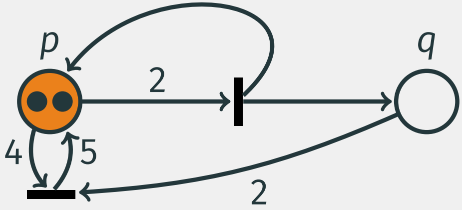



$(2 \;\; 0)$ can cover $(0 \;\; 2)$?

$$2 + \quad x - y = 0$$

$$0 - \; 2x + y = 2$$

# Over-approximating coverability: trap constraints

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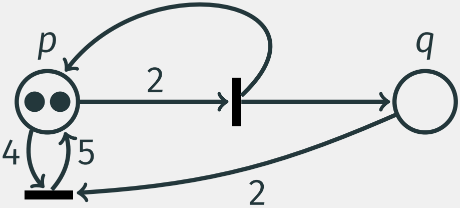
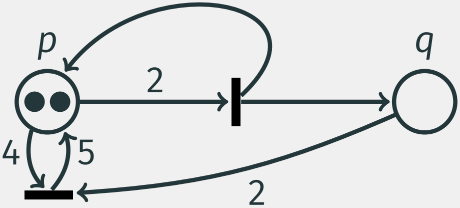



$(2 \;\; 0)$ can cover $(0 \;\; 2)$?

State equation ✓

Trap constraints ✗

## Over-approximating coverability: trap constraints

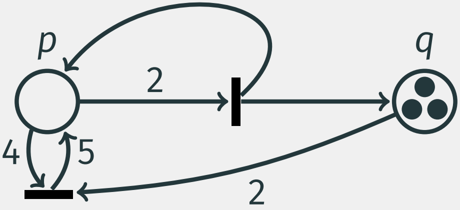



$(2 \;\; 0)$ can cover $(0 \;\; 2)$? *No!*

State equation ✓

Trap constraints ✗

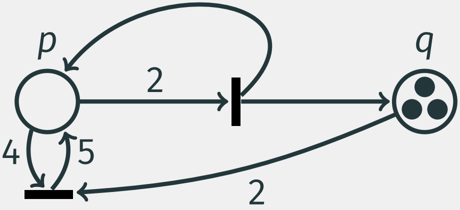
# Over-approximating coverability: trap constraints




$\begin{pmatrix}0 & 3\end{pmatrix}$ can cover $\begin{pmatrix}1 & 1\end{pmatrix}$?

$$0 + \quad x - y \geq 1$$
$$3 - 2x + y \geq 1$$

# Over-approximating coverability: trap constraints

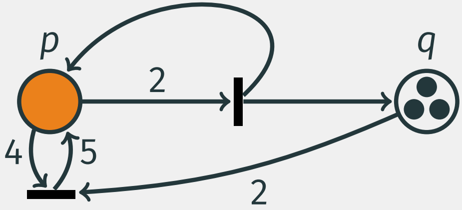



$(0 \;\; 3)$ can cover $(1 \;\; 1)$?

$$0 + \quad 1 - 0 \geq 1$$

$$3 - 2 \cdot 1 + 0 \geq 1$$

# Over-approximating coverability: trap constraints

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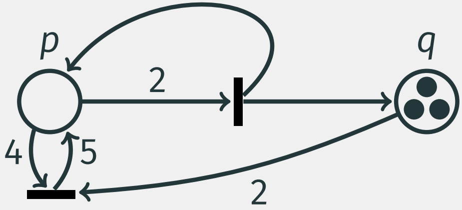


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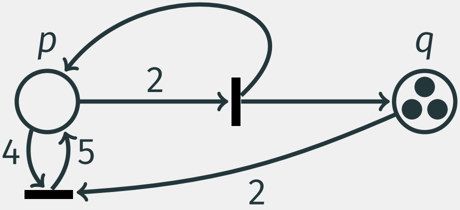
State equation ✓

Trap constraints ✓

# Over-approximating coverability: trap constraints




(0  3) can cover (1  1)?

Yet, not coverable...

State equation ✓

Trap constraints ✓

$\boldsymbol{m}$ is coverable from $\boldsymbol{m}_0$

$$\Downarrow$$

$\boldsymbol{m}_0, \boldsymbol{m}$ satisfy

state equation and trap constraints

# Deciding safety with over-approximations

$\boldsymbol{m}$ is not coverable from $\boldsymbol{m}_0$

Safety

$\Uparrow$

$\boldsymbol{m}_0, \boldsymbol{m}$ do not satisfy

state equation or trap constraints

$\boldsymbol{m}$ is not coverable from $\boldsymbol{m}_0$

$\Uparrow$

$\boldsymbol{m}_0, \boldsymbol{m}$ do not satisfy

state equation or trap constraints

Efficient in practice!

Esparza, Ledesma-Garza, Majumdar, Meyer & Niksic CAV'14

$\boldsymbol{m}$ is not coverable from $\boldsymbol{m}_0$

$$\Uparrow$$

$\boldsymbol{m}_0, \boldsymbol{m}$ do not satisfy

state equation or trap constraints

Any finer approximation, yet efficient?

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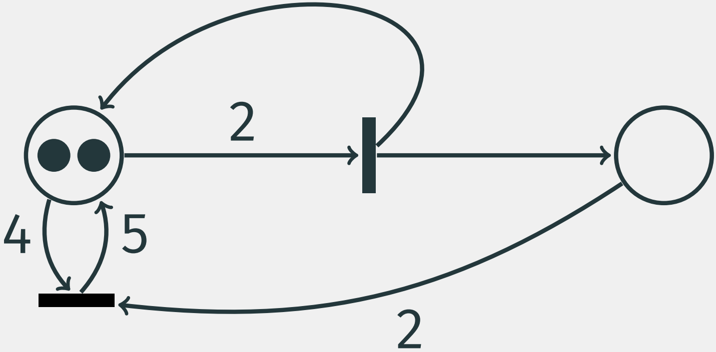

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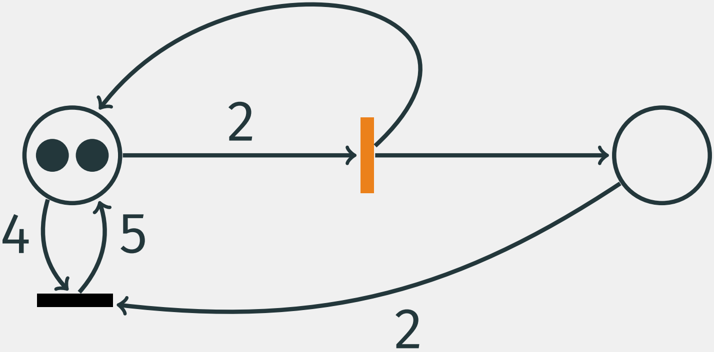

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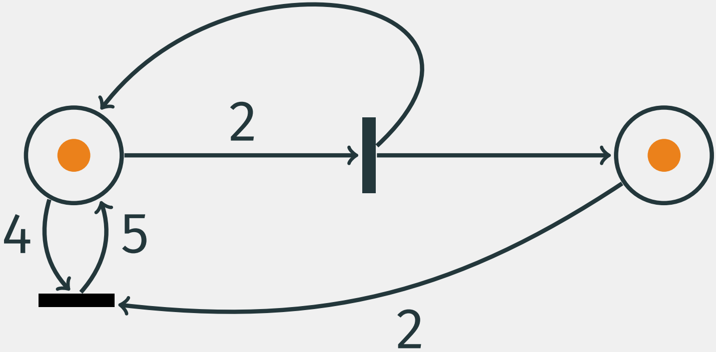

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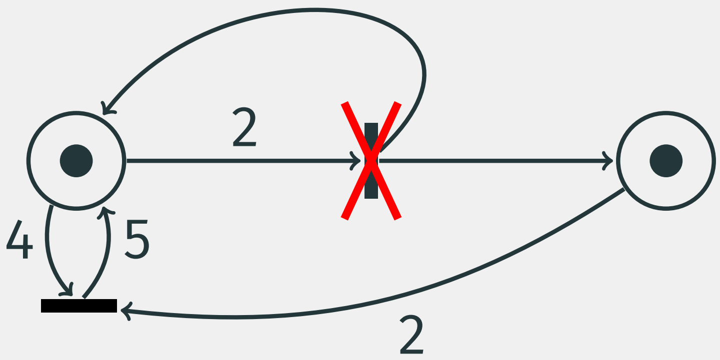
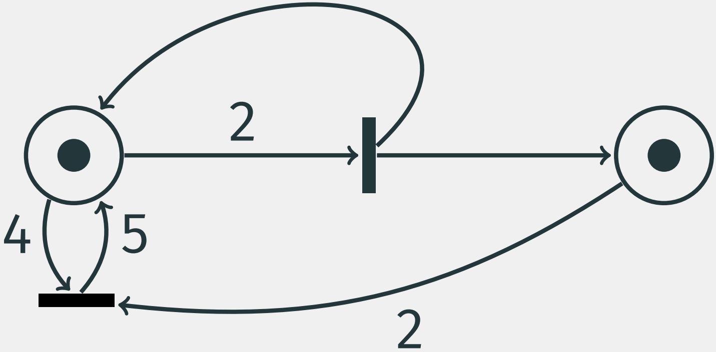
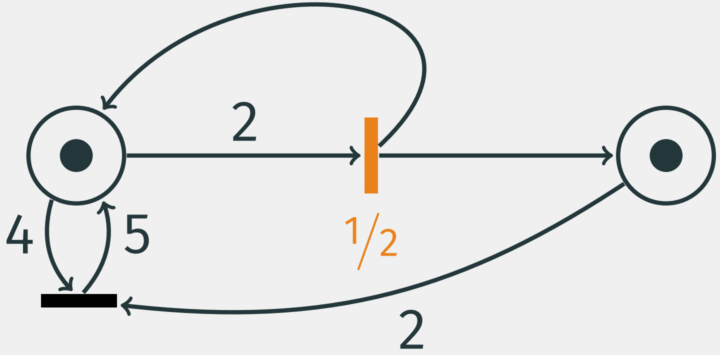
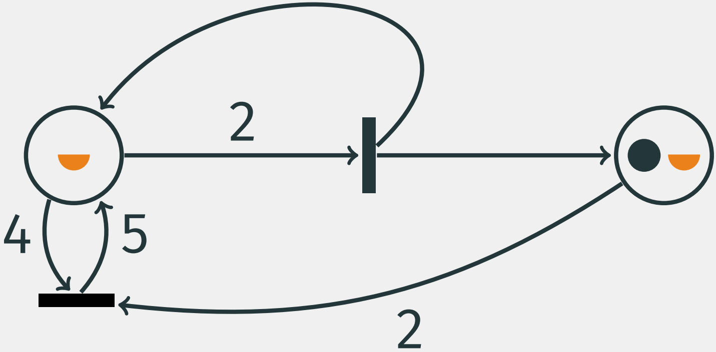


# ~~(Discrete)~~ Continuous Petri nets



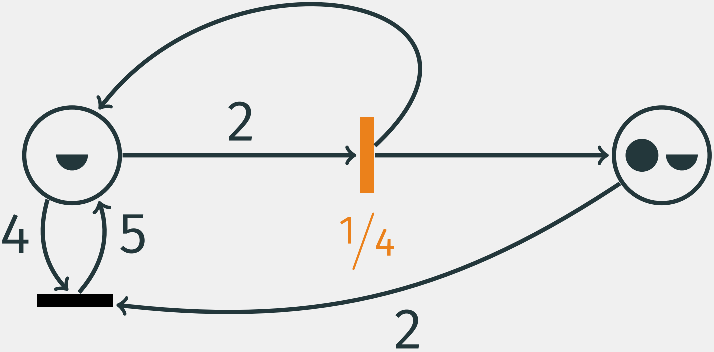


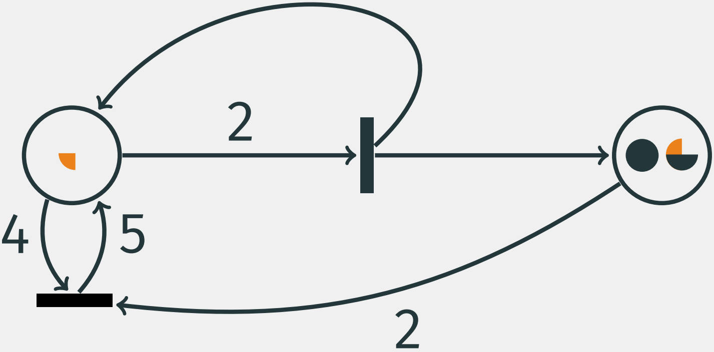

2
4
5
1/2
2

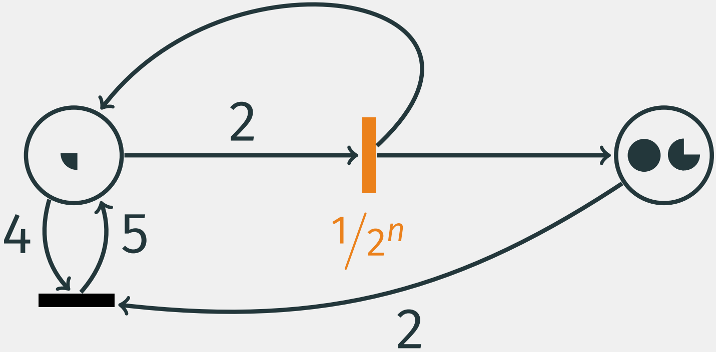

2
4
5
2

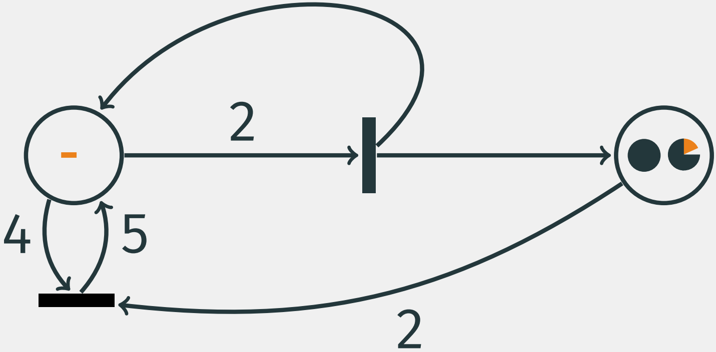
# ~~(Discrete)~~ Continuous Petri nets

David & Alla PN'87




2
4
5
2


2
4
5
1/2^n
2


2
4
5
2

# Continuity to over-approximate coverability

$$\boldsymbol{m} \text{ is coverable from } \boldsymbol{m}_0$$

$$\Downarrow$$

$$\boldsymbol{m} \text{ is } \mathbb{Q}\text{-coverable from } \boldsymbol{m}_0$$

# Continuity to over-approximate coverability

$\boldsymbol{m}$ is coverable from $\boldsymbol{m}_0$

EXPSPACE

$$\Downarrow$$

$\boldsymbol{m}$ is $\mathbb{Q}$-coverable from $\boldsymbol{m}_0$

PTIME

$$\Downarrow \quad \not\Uparrow$$

$\boldsymbol{m}_0$ and $\boldsymbol{m}$ satisfy
state equation & trap constraints

Esparza & Melzer FMSD'00
Esparza, Ledesma-Garza, Majumdar, Meyer & Niksic CAV'14

PTIME / NP / coNP

# Continuity to over-approximate coverability

$\boldsymbol{m}$ is not coverable from $\boldsymbol{m}_0$

Safety

$\Uparrow$

$\boldsymbol{m}$ is not $\mathbb{Q}$-coverable from $\boldsymbol{m}_0$

# Coverability in continuous Petri nets

Fix some continuous Petri net $(P, T, \mathbf{Pre}, \mathbf{Post})$

**$m$ is $\mathbb{Q}$-coverable from $m_0$ iff...** **Fraca & Haddad** Fundam. Inf.'15

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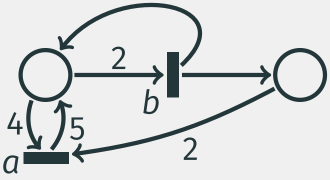


$$\boldsymbol{m}_0 = (0, 3)$$
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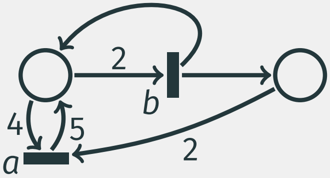


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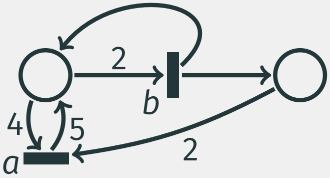


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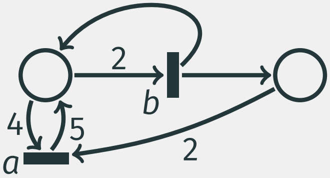


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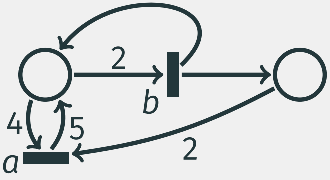


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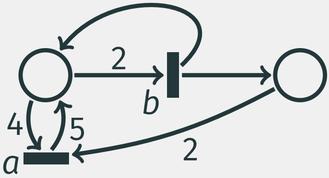


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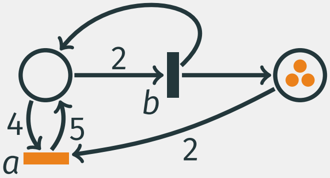


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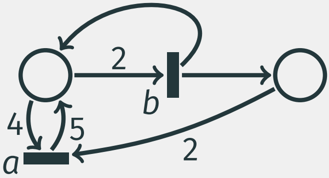


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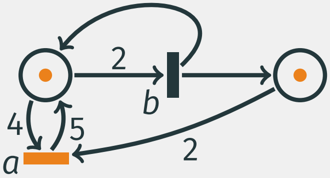


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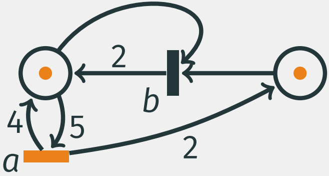


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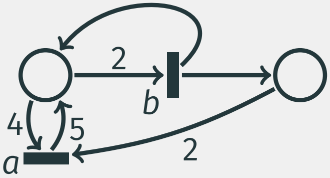


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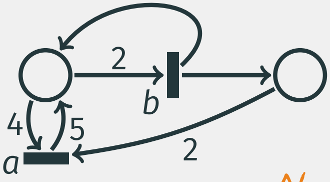


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Not ℚ-coverable from

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# Coverability in continuous Petri nets

Polynomial time!

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# Coverability in continuous Petri nets

**Logical characterization** — B., Finkel, Haase & Haddad TACAS'16

$\mathbb{Q}$-coverability can be encoded in a linear size formula of

existential FO$(\mathbb{Q}_{\geq 0}, +, <)$

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↖ Even better approximation

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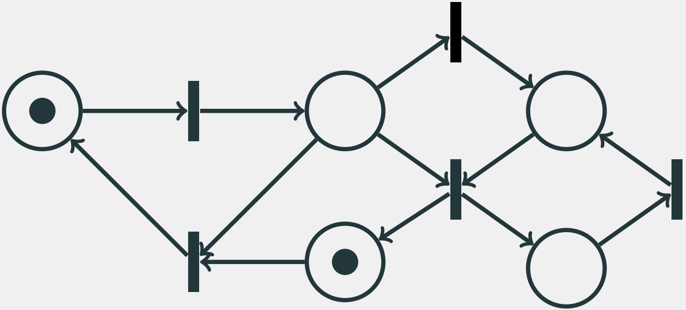
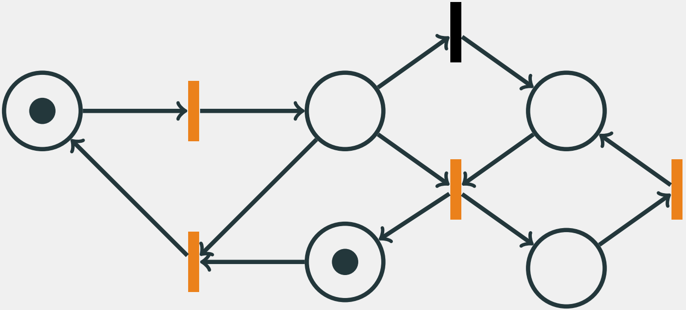
More subtle

# Encoding the firing set conditions



Testing whether some transitions can be fired from initial marking

# Encoding the firing set conditions



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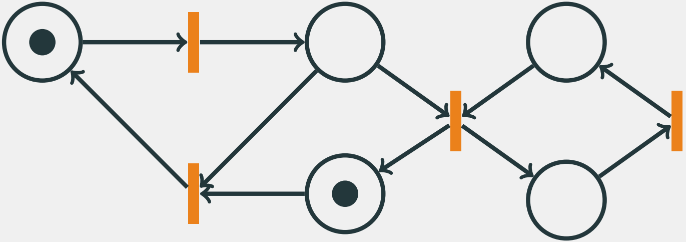

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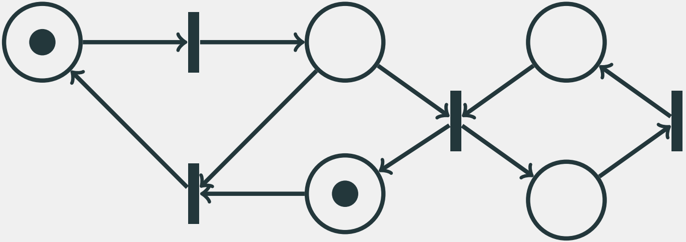
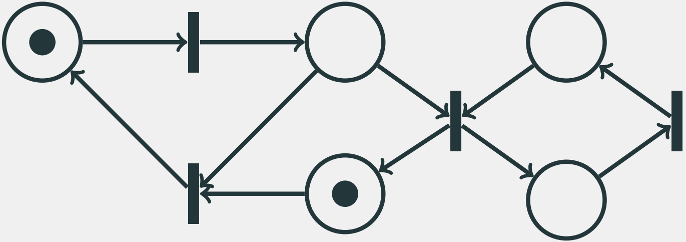


Simulate a "breadth-first" transitions firing

# Encoding the firing set conditions



Simulate a "breadth-first" transitions firing
by numbering places/transitions

Verma, Seidl & Schwentick CADE'05

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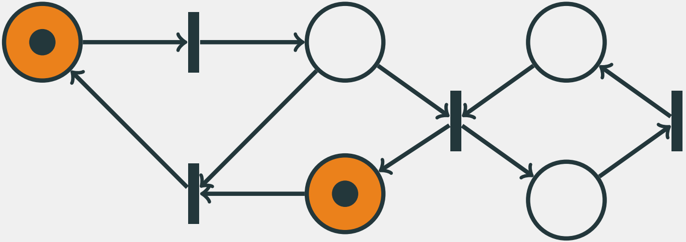

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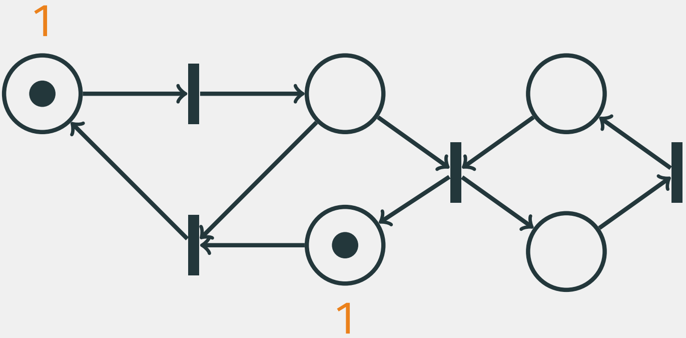


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Verma, Seidl & Schwentick CADE'05

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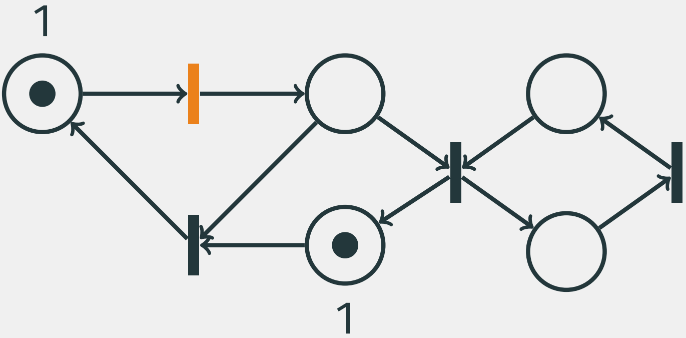


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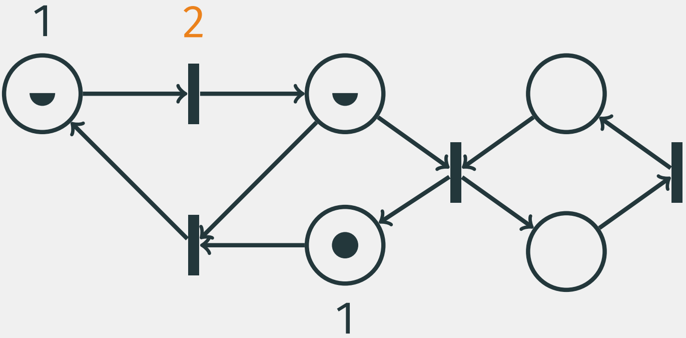


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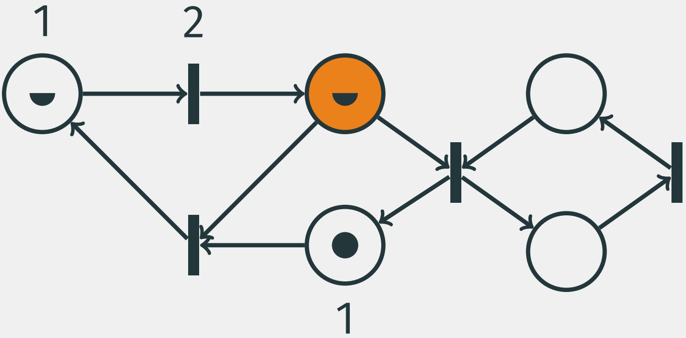


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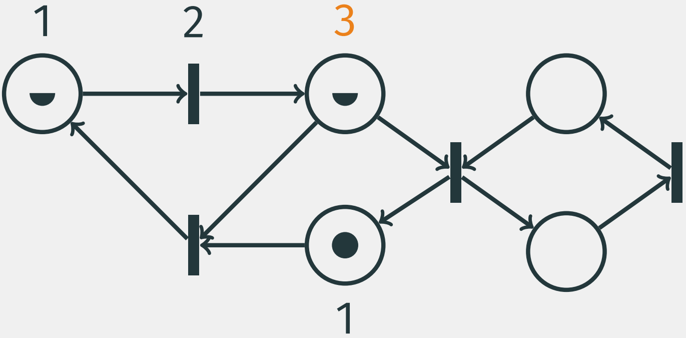


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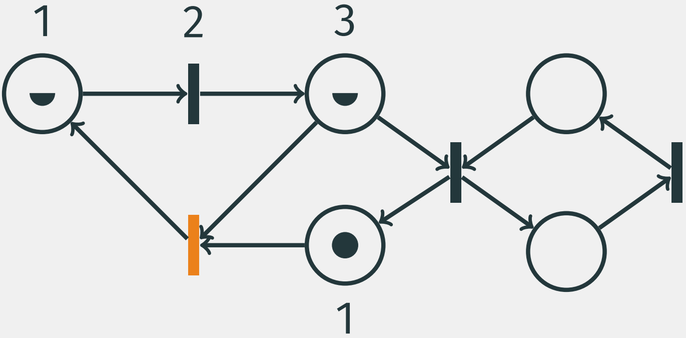


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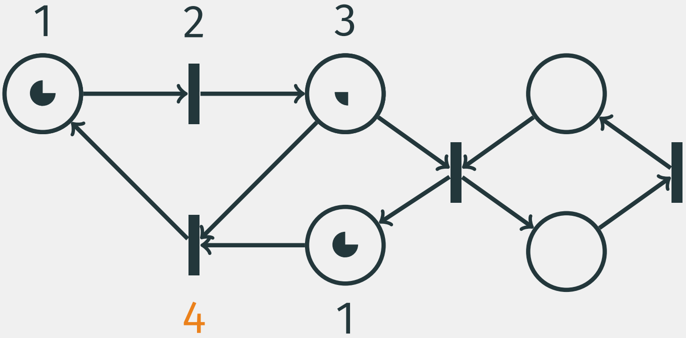


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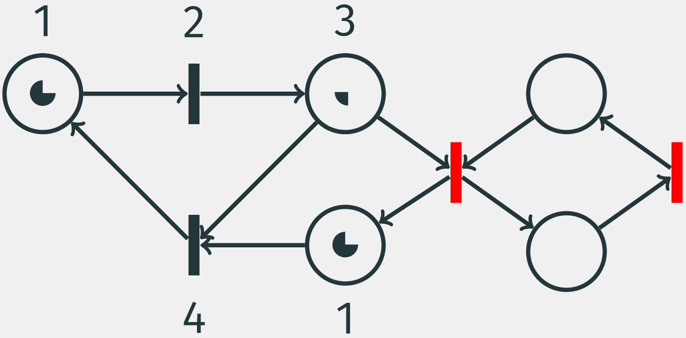


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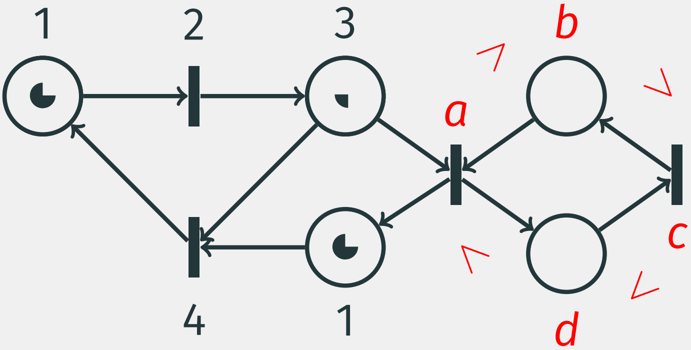


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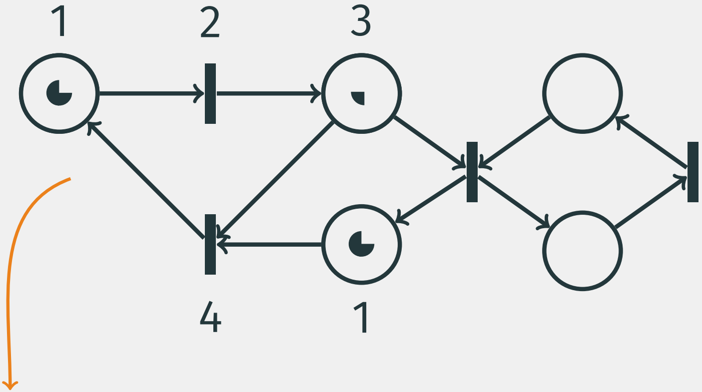


$$\varphi(\boldsymbol{x}) = \exists \boldsymbol{y} : \bigwedge_{p \in P} \boldsymbol{y}(p) > 0 \rightarrow \bigwedge_{t \in {}^\bullet p} \boldsymbol{y}(t) < \boldsymbol{y}(p) \cdots$$

$\mathbb{Q}$-coverability: efficient but incomplete...

Combine approaches!

# From $\mathbb{Q}$-coverability to a complete algorithm

**Forward algorithm** **Karp & Miller** JCSS'69

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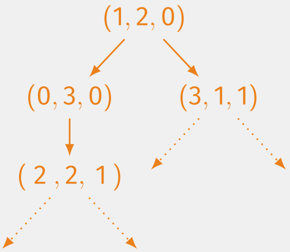

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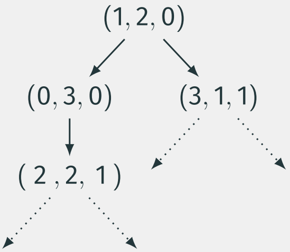

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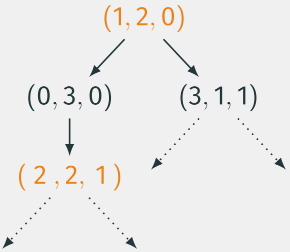

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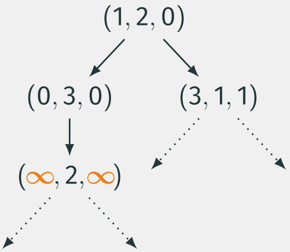

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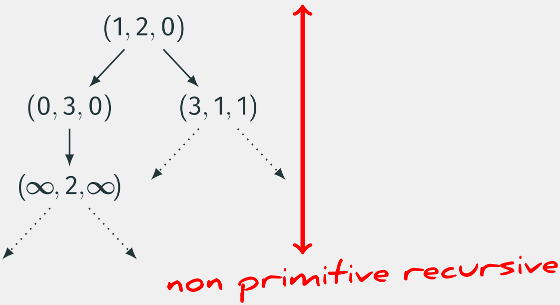

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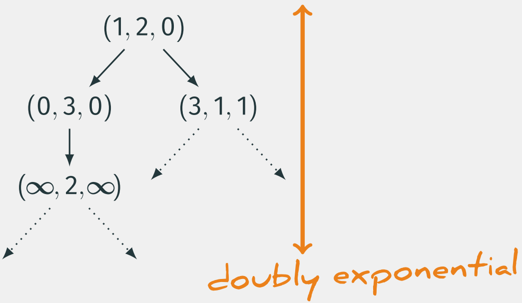

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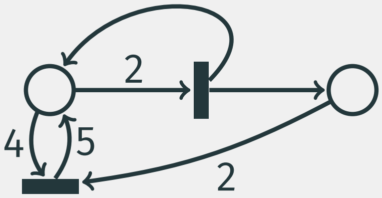
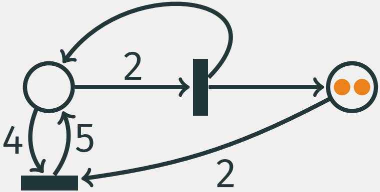
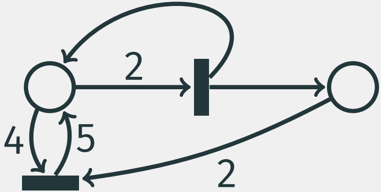
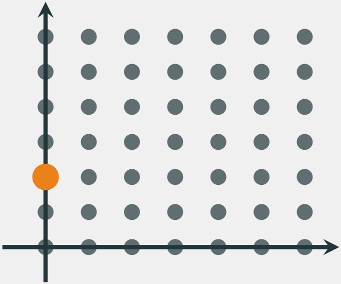





What initial markings may cover $(0, 2)$?

# Backward algorithm



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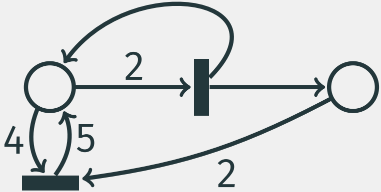


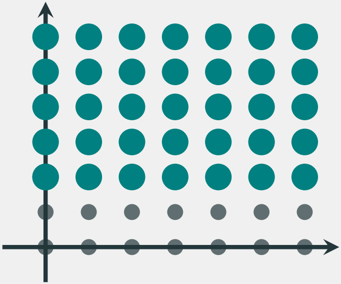

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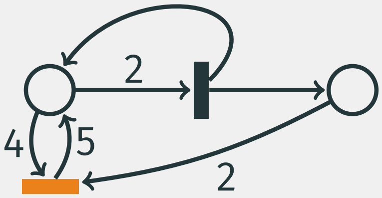


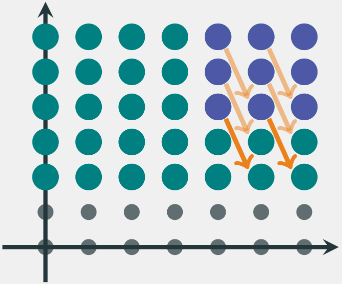

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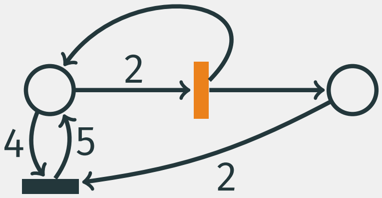


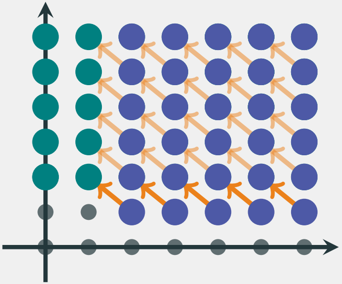

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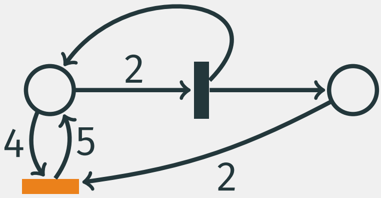


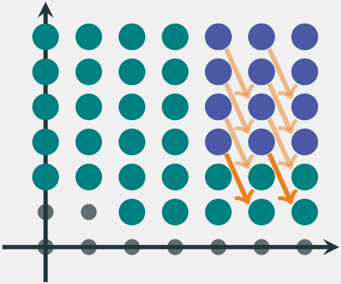

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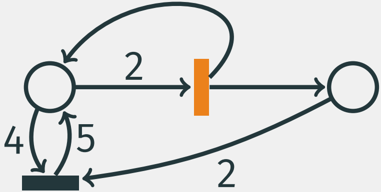


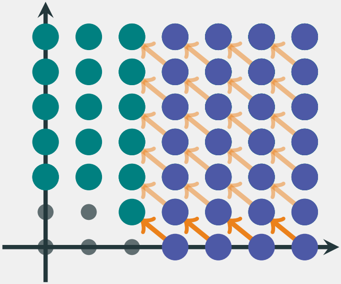

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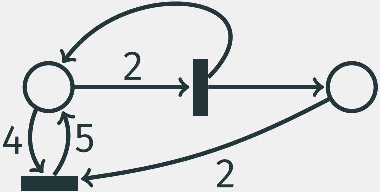


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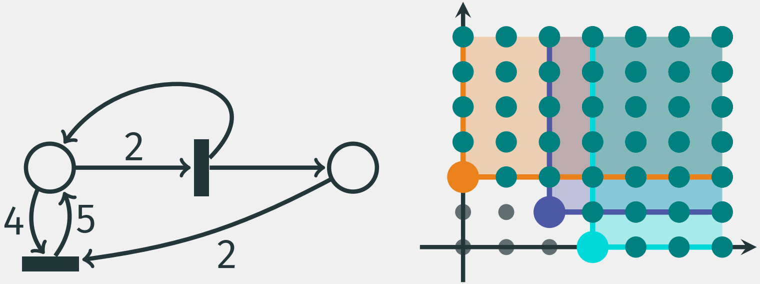
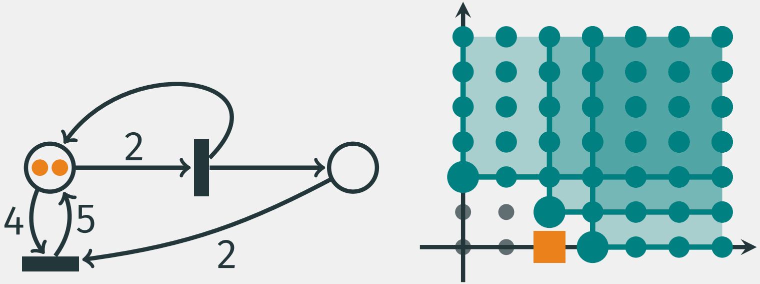



Basis size may become doubly exponential

Bozzelli & Ganty RP'11

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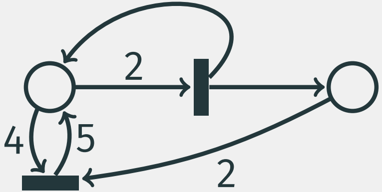


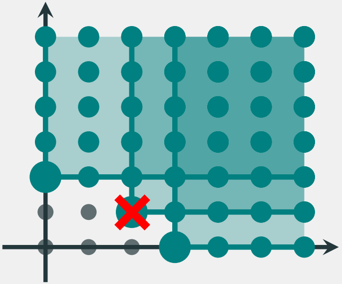

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Prune basis with $\mathbb{Q}$-coverability!

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```
if target marking m is not ℚ-coverable:
  return False
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Polynomial time

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$X = \{\text{target marking } \boldsymbol{m}\}$

`while` (initial marking $\boldsymbol{m}_0$ not covered by $X$):

  $B =$ markings obtained from $X$ one step backward

  $B = B \setminus \{\boldsymbol{b} \in B : \neg\varphi(\boldsymbol{b})\}$

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  $\varphi(\boldsymbol{x}) = \varphi(\boldsymbol{x}) \ \wedge \ \bigwedge_{\text{pruned } \boldsymbol{b}} \boldsymbol{x} \not\geq \boldsymbol{b}$

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- supports dense/sparse matrices through NUMPY/SCIPY
- experimental parallelism support

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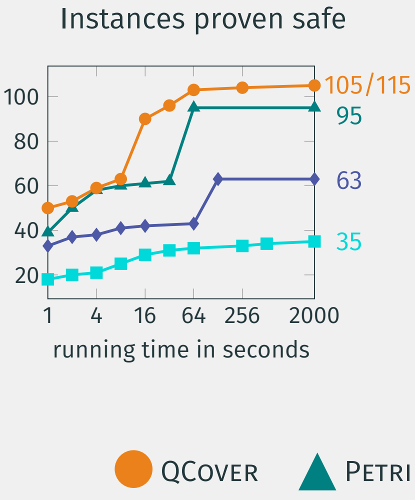
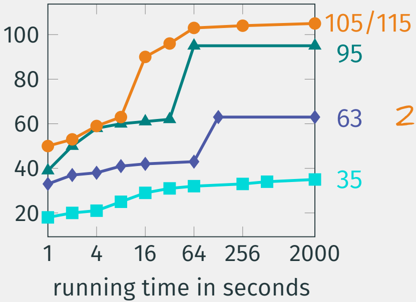


Instances proven safe
100
80
60
40
20
1
4
16
64
256
2000
running time in seconds
105/115
95
63
35
QCover
Petrinizer
BFC
MIST

# Benchmarks

Instances proven safe




Largest nets proved safe:

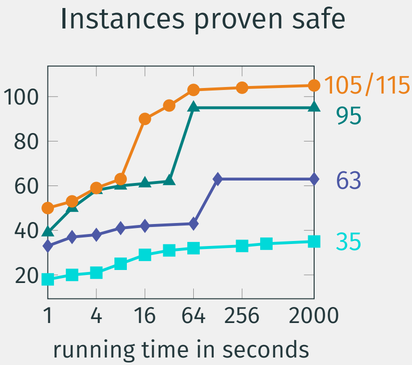
21143 places
7150 trans.
42 secs.

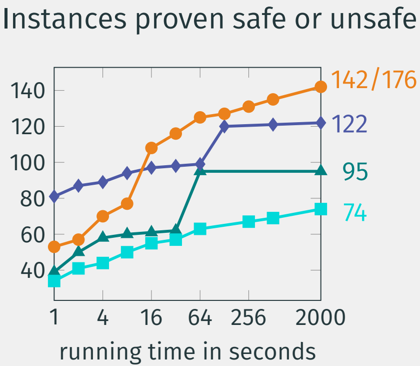
6690 places
11934 trans.
21 secs.

754 places
27370 trans.
3 secs.

QCover PETRINIZER BFC MIST

# Benchmarks

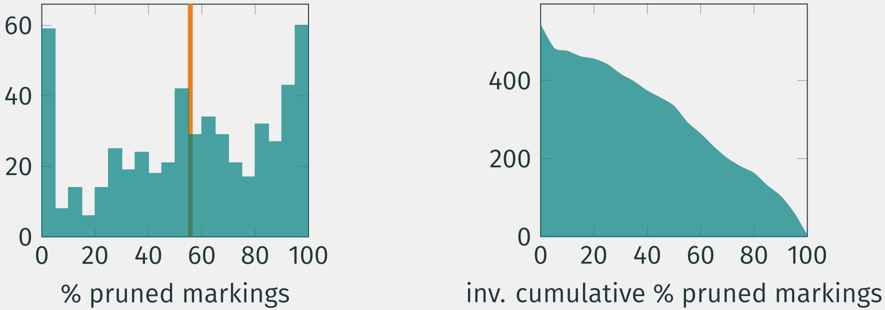

Instances proven safe
100
80
60
40
20
1
4
16
64
256
2000
running time in seconds
105/115
95
63
35


Instances proven safe or unsafe
140
120
100
80
60
40
1
4
16
64
256
2000
running time in seconds
142/176
122
95
74


QCover
Petrinizer
BFC
MIST

# Benchmarks

Markings pruning efficiency across all iterations



# Future work

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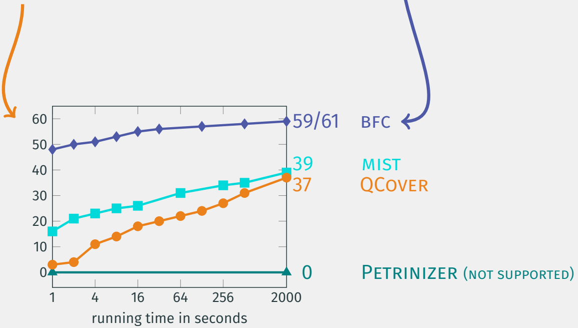

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- Combine our approach with a forward algorithm to better handle unsafe instances
- Use more efficient data structures, *e.g.* sharing trees

  (Delzanno, Raskin & Van Begin STTT'04)

# Future work

- Combine our approach with a forward algorithm to better handle unsafe instances
- Use more efficient data structures, *e.g.* sharing trees

  (Delzanno, Raskin & Van Begin STTT'04)
- Extend to Petri nets with transfer/reset arcs

# Thank you!
# Vielen Dank!