The Logical View on Continuous Petri Nets

Michael Blondin

Joint work with Alain Finkel, Christoph Haase, Serge Haddad

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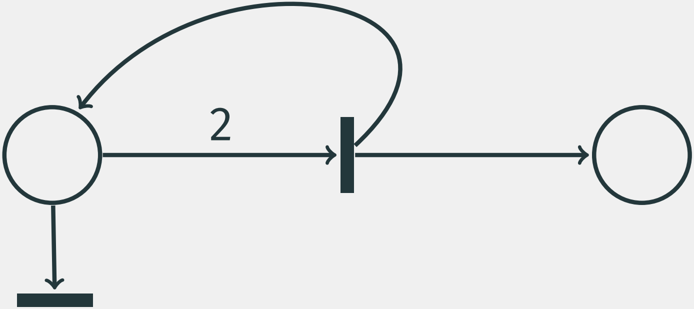
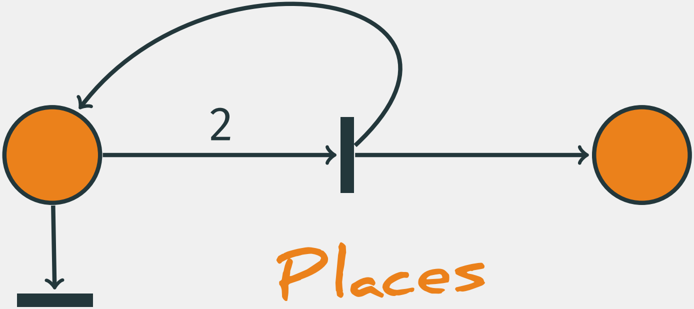
Petri nets

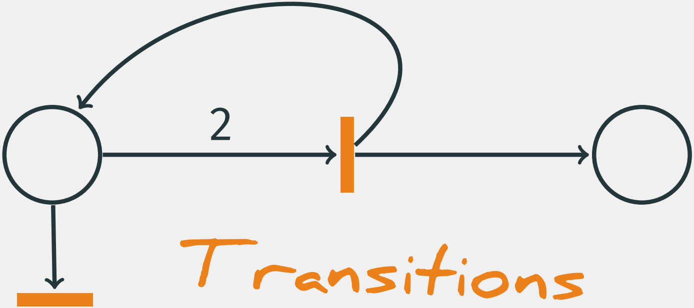


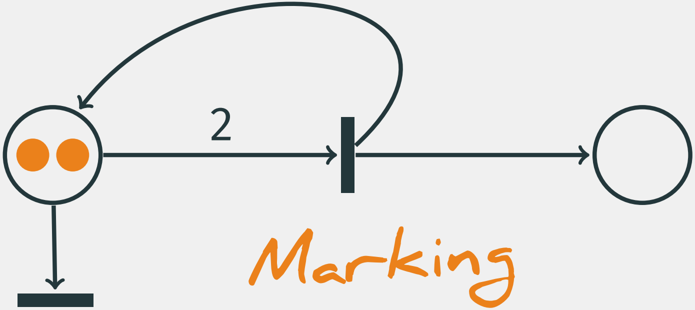
Petri nets



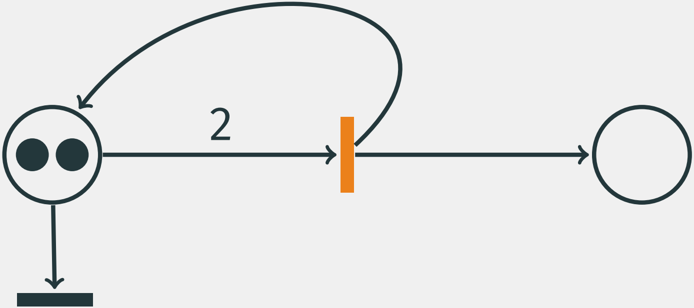
Places

Petri nets

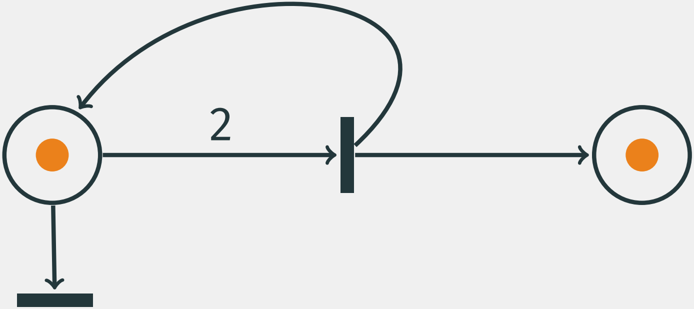



2
Marking

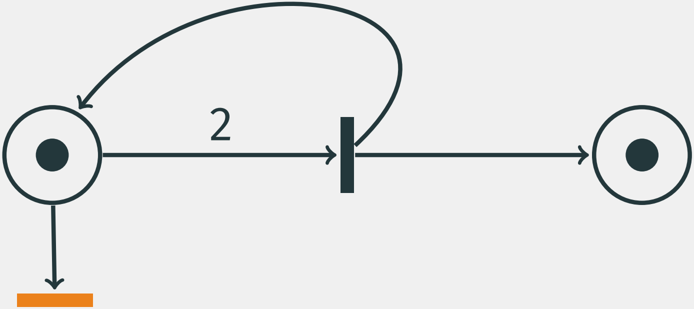
Petri nets



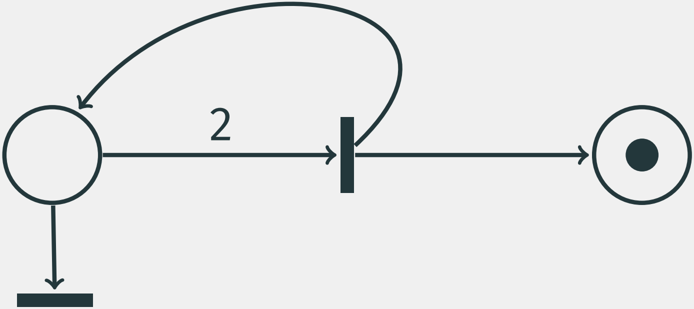
Petri nets



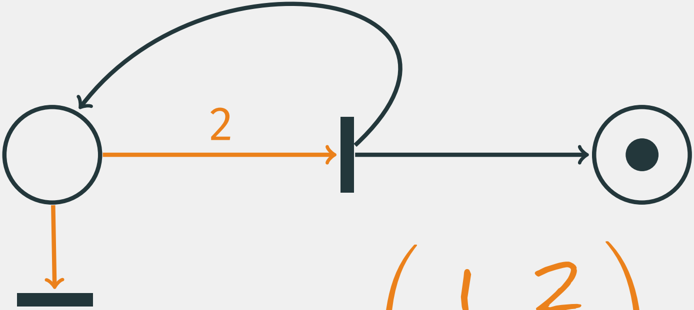
Petri nets



Petri nets

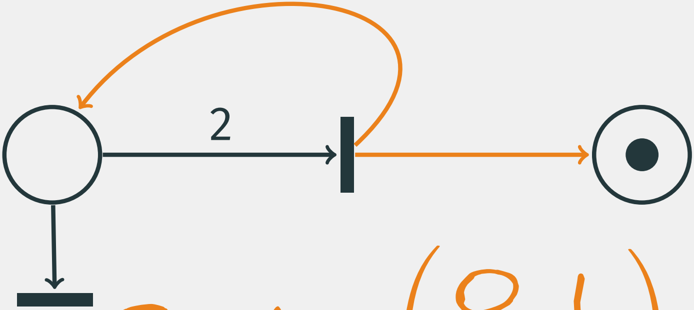


Petri nets



$$Pre = \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix}$$

Petri nets



$$\mathrm{Post} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$$

Safety verification with Petri nets

Process 1

Process 2

Safety verification with Petri nets

Process 1

critical section

Process 2

critical section

Safety verification with Petri nets

```
while true:
  x = true
  while y: pass
  # critical section
  x = false
```

```
while true:
  y = true
  if x then:
    y = false
    while x: pass
    goto
  # critical section
  y = false
```

Lamport mutual exclusion algorithm

Safety verification with Petri nets

```
while true:
  x = true
  while y: pass
  # critical section
  x = false
```

```
  while true:
★   y = true
    if x then:
      y = false
      while x: pass
      goto ★
    # critical section
    y = false
```

shared variables: x, y

Safety verification with Petri nets

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while true:
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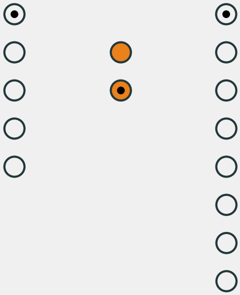

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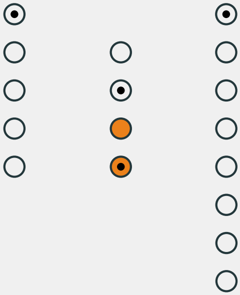

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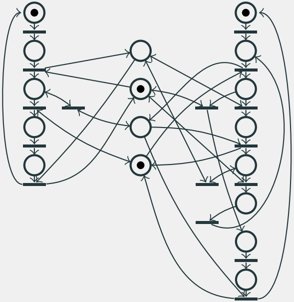

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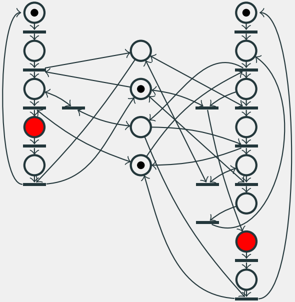

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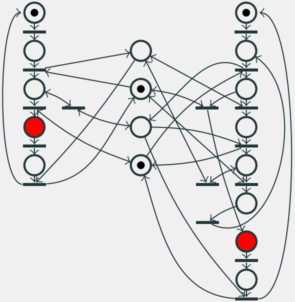
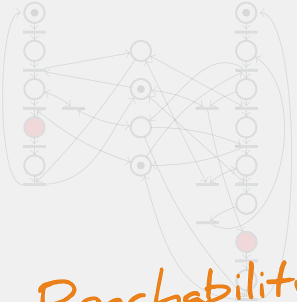
Safety verification with Petri nets



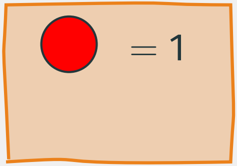
Processes at both critical sections $\iff$  $= 1$

Safety verification with Petri nets

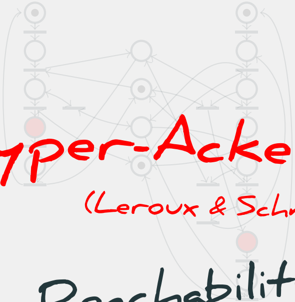


Reachability problem

Processes at both critical sections $\Longleftrightarrow$



Safety verification with Petri nets

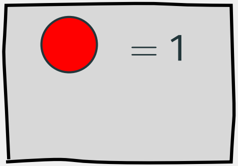


Hyper-Ackermannian!

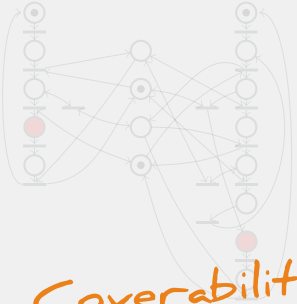
(Leroux & Schmitz '15)

Reachability problem

Processes at both critical sections $\iff$

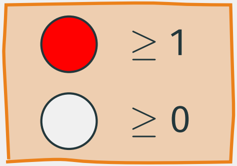


Safety verification with Petri nets



Coverability problem

Processes at both critical sections $\iff$

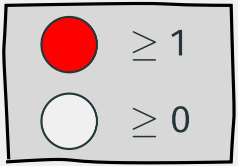


EXPSPACE-complete!

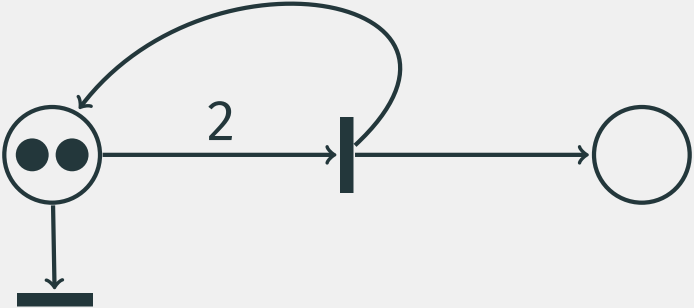
(Lipton '76, Rackoff '78)

Coverability problem

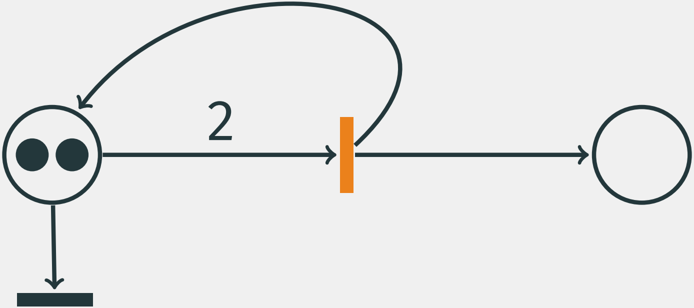
Processes at both critical sections $\iff$



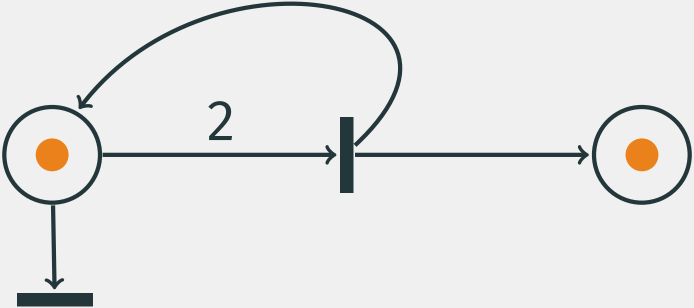
(Discrete) Petri nets



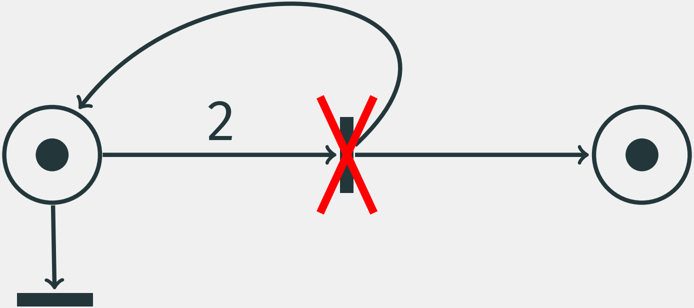
(Discrete) Petri nets



(Discrete) Petri nets

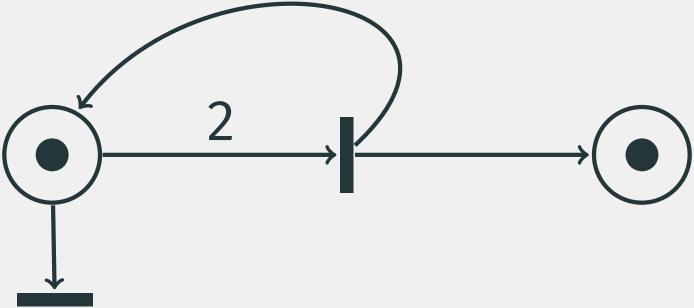


(Discrete) Petri nets



~~(Discrete)~~ Continuous Petri nets

David & Alla '87



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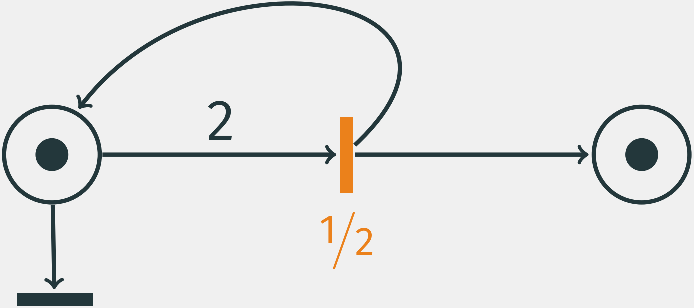

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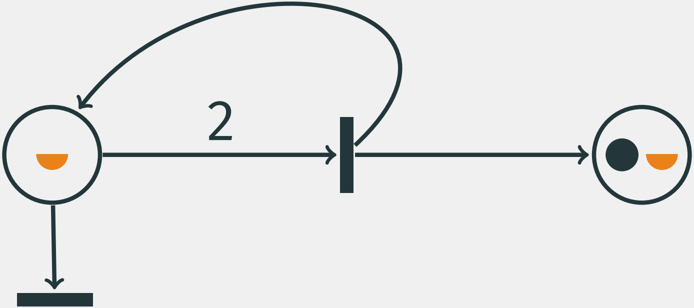

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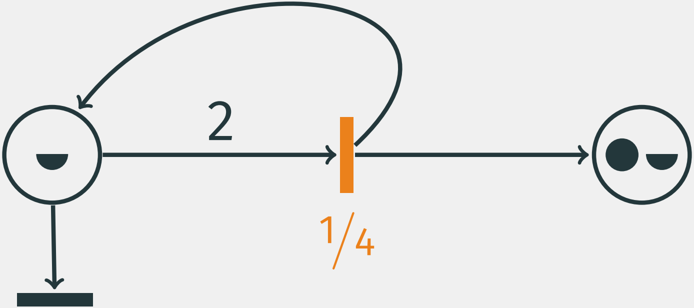

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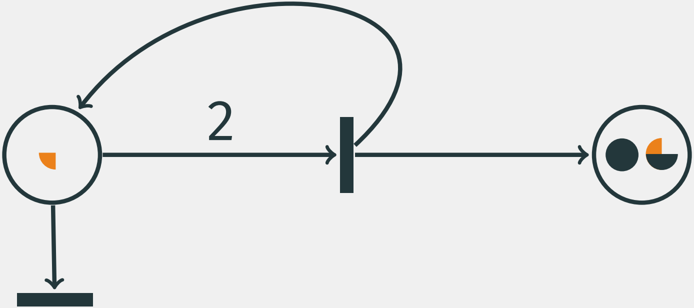

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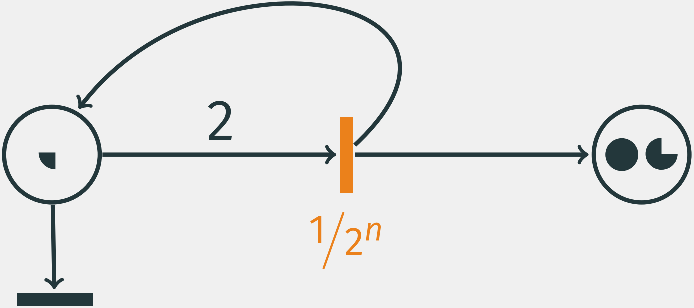

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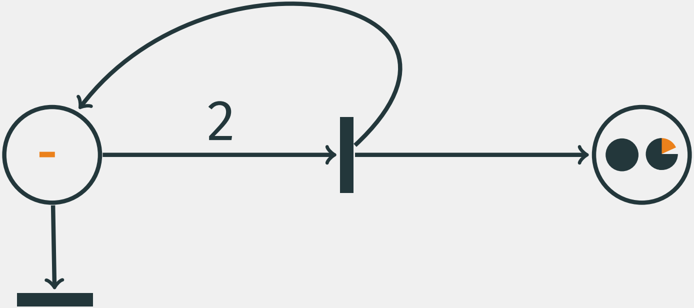

$$\boldsymbol{m}_{\text{target}} \text{ reachable from } \boldsymbol{m}_{\text{init}}$$

$$\Downarrow$$

$$\boldsymbol{m}_{\text{target}}\ \mathbb{Q}\text{-reachable from } \boldsymbol{m}_{\text{init}}$$

$\boldsymbol{m}_{\text{target}}$ not reachable from $\boldsymbol{m}_{\text{init}}$

Safety

$\Uparrow$

$\boldsymbol{m}_{\text{target}}$ not $\mathbb{Q}$-reachable from $\boldsymbol{m}_{\text{init}}$

$$\boldsymbol{m}_{\text{target}} \text{ not reachable from } \boldsymbol{m}_{\text{init}}$$

Safety

$$\Uparrow$$

$$\boldsymbol{m}_{\text{target}} \text{ not } \mathbb{Q}\text{-reachable from } \boldsymbol{m}_{\text{init}}$$

Q-reachability $\in$ PTIME!

(Fraca & Haddad '13)

Logical characterization of $\mathbb{Q}$-reachability

Fix some continuous Petri net $(P, T, \mathbf{Pre}, \mathbf{Post})$

m_{init} is $\mathbb{Q}$-reachable from m_{target} iff... **Fraca & Haddad '13**

Logical characterization of $\mathbb{Q}$-reachability

Fix some continuous Petri net $(P, T, \mathbf{Pre}, \mathbf{Post})$

$\boldsymbol{m}_{\text{init}}$ is $\mathbb{Q}$-reachable from $\boldsymbol{m}_{\text{target}}$ iff... **Fraca & Haddad '13**

there exists $\boldsymbol{v} \in \mathbb{Q}_{\geq 0}^{T}$ such that

- $\boldsymbol{m}_{\text{target}} = \boldsymbol{m}_{\text{init}} + (\mathbf{Post} - \mathbf{Pre}) \cdot \boldsymbol{v}$

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there exists $\boldsymbol{v} \in \mathbb{Q}^{T}_{\geq 0}$ such that

- $\boldsymbol{m}_{\text{target}} = \boldsymbol{m}_{\text{init}} + (\mathbf{Post} - \mathbf{Pre}) \cdot \boldsymbol{v}$
- an execution from $\boldsymbol{m}_{\text{init}}$ fires exactly $\{t \in T : \boldsymbol{v}_t > 0\}$

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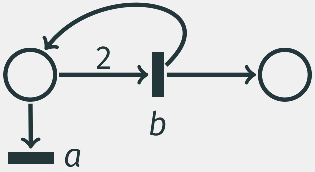
Logical characterization of $\mathbb{Q}$-reachability



$$\boldsymbol{m}_{\text{init}} = (2, 0)$$
$$\boldsymbol{m}_{\text{target}} = (0, 2)$$

$\boldsymbol{m}_{\text{init}}$ is $\mathbb{Q}$-reachable from $\boldsymbol{m}_{\text{target}}$ iff... **Fraca & Haddad '13**

there exists $\boldsymbol{v}_a, \boldsymbol{v}_b \in \mathbb{Q}_{\geq 0}$ such that

- $\boldsymbol{m}_{\text{target}} = \boldsymbol{m}_{\text{init}} + (\mathbf{Post} - \mathbf{Pre}) \cdot \boldsymbol{v}$
- an execution from $\boldsymbol{m}_{\text{init}}$ fires exactly $\{t \in \{a, b\} : \boldsymbol{v}_t > 0\}$
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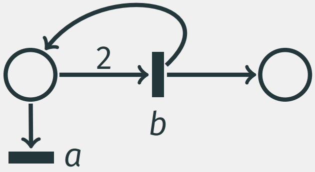
Logical characterization of $\mathbb{Q}$-reachability



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- $$\begin{aligned} 2 - \boldsymbol{v}_a - \boldsymbol{v}_b &= 0 \\ \boldsymbol{v}_b &= 2 \end{aligned}$$
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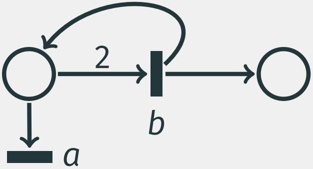

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- $\begin{aligned} 2 - \boldsymbol{v}_a - \boldsymbol{v}_b &= 0 \\ \boldsymbol{v}_b &= 2 \end{aligned} \implies \boldsymbol{v}_a = 0,\ \boldsymbol{v}_b = 2$
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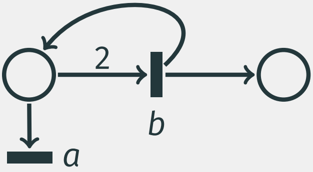

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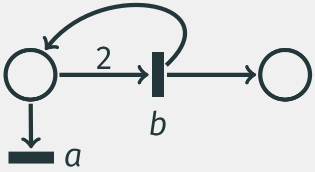

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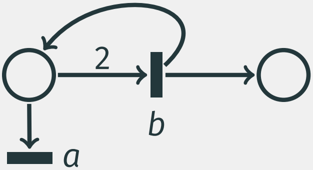

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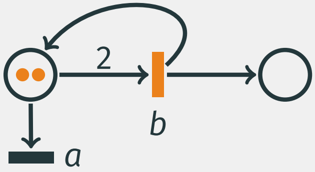
there exists $\boldsymbol{v}_a, \boldsymbol{v}_b \in \mathbb{Q}_{\geq 0}$ such that

- $\begin{array}{r} 2 - \boldsymbol{v}_a - \boldsymbol{v}_b = 0 \\ \boldsymbol{v}_b = 2 \end{array} \implies \boldsymbol{v}_a = 0,\ \boldsymbol{v}_b = 2$ ✓
- an execution from $\boldsymbol{m}_{\text{init}}$ fires exactly $\{b\}$
- an execution to $\boldsymbol{m}_{\text{target}}$ fires exactly $\{b\}$

Logical characterization of $\mathbb{Q}$-reachability



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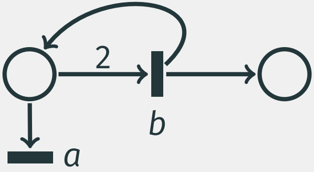
Logical characterization of $\mathbb{Q}$-reachability



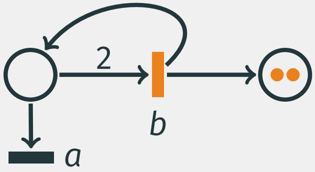
$$\begin{aligned} \boldsymbol{m}_{\text{init}} &= (2, 0) \\ \boldsymbol{m}_{\text{target}} &= (0, 2) \end{aligned}$$

$\boldsymbol{m}_{\text{init}}$ is $\mathbb{Q}$-reachable from $\boldsymbol{m}_{\text{target}}$ iff... **Fraca & Haddad '13**

there exists $\boldsymbol{v}_a, \boldsymbol{v}_b \in \mathbb{Q}_{\geq 0}$ such that

- $\begin{aligned} 2 - \boldsymbol{v}_a - \boldsymbol{v}_b &= 0 \\ \boldsymbol{v}_b &= 2 \end{aligned} \implies \boldsymbol{v}_a = 0,\ \boldsymbol{v}_b = 2$ ✓
- an execution from $\boldsymbol{m}_{\text{init}}$ fires exactly $\{b\}$ ✓
- an execution to $\boldsymbol{m}_{\text{target}}$ fires exactly $\{b\}$

Logical characterization of $\mathbb{Q}$-reachability



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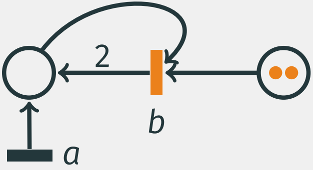

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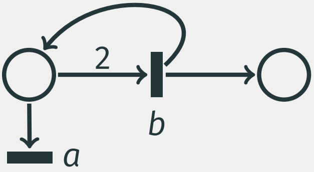

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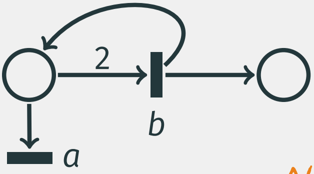
Logical characterization of $\mathbb{Q}$-reachability



$\boldsymbol{m}_{\text{init}} = (2, 0)$

$\boxed{\boldsymbol{m}_{\text{target}} = (0, 2)}$

Not Q-reachable from

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Logical characterization of $\mathbb{Q}$-reachability

Theorem — **B., Finkel, Haase & Haddad '16**

$\mathbb{Q}$-reachability definable by linear size formula of

$$\exists\, \mathsf{FO}(\mathbb{Q}, +, <)$$

$\boldsymbol{m}_{\text{init}}$ is $\mathbb{Q}$-reachable from $\boldsymbol{m}_{\text{target}}$ iff... — **Fraca & Haddad '13**

there exists $\boldsymbol{v} \in \mathbb{Q}_{\geq 0}^{T}$ such that

- $\boldsymbol{m}_{\text{target}} = \boldsymbol{m}_{\text{init}} + (\mathbf{Post} - \mathbf{Pre}) \cdot \boldsymbol{v}$
- an execution from $\boldsymbol{m}_{\text{init}}$ fires exactly $\{t \in T : \boldsymbol{v}_t > 0\}$
- an execution to $\boldsymbol{m}_{\text{target}}$ fires exactly $\{t \in T : \boldsymbol{v}_t > 0\}$

Logical characterization of $\mathbb{Q}$-reachability

Theorem — B., Finkel, Haase & Haddad '16

$\mathbb{Q}$-reachability definable by linear size formula of

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Better approximation!

$\boldsymbol{m}_{\text{init}}$ is $\mathbb{Q}$-reachable from $\boldsymbol{m}_{\text{target}}$ iff... — Fraca & Haddad '13

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Testing validity $\in$ NP

$\boldsymbol{m}_{\text{init}}$ is $\mathbb{Q}$-reachable from $\boldsymbol{m}_{\text{target}}$ iff... — Fraca & Haddad '13

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Logical characterization of $\mathbb{Q}$-reachability

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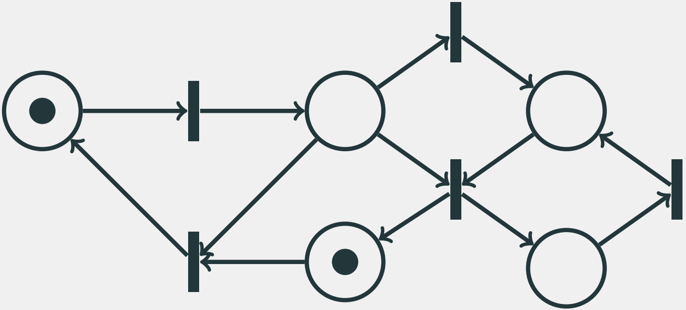
there exists $\boldsymbol{v} \in \mathbb{Q}_{\geq 0}^{T}$ such that

- $\boldsymbol{m}_{\text{target}} = \boldsymbol{m}_{\text{init}} + (\mathbf{Post} - \mathbf{Pre}) \cdot \boldsymbol{v}$

More subtle

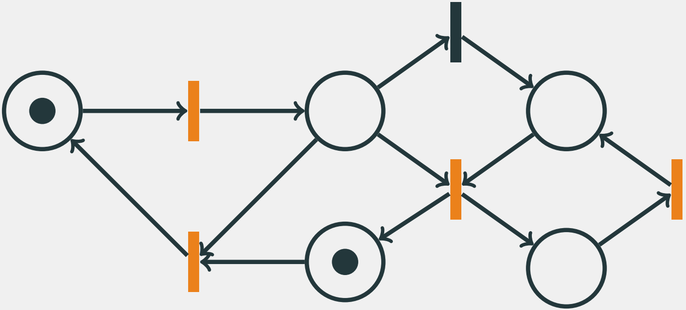
- an execution from $\boldsymbol{m}_{\text{init}}$ fires exactly $\{t \in T : \boldsymbol{v}_t > 0\}$
- an execution to $\boldsymbol{m}_{\text{target}}$ fires exactly $\{t \in T : \boldsymbol{v}_t > 0\}$

Encoding the firing set conditions



Testing whether some transitions can be fired from initial marking

Encoding the firing set conditions



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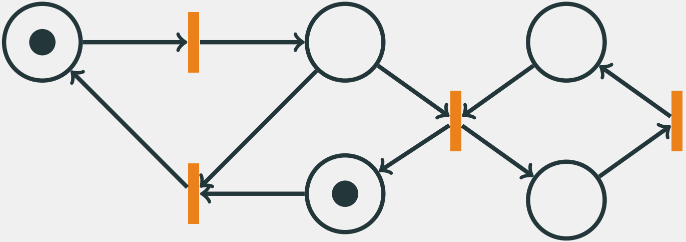

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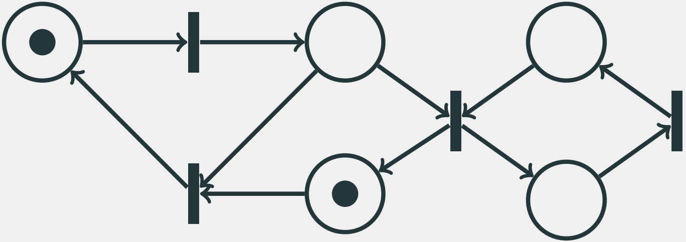
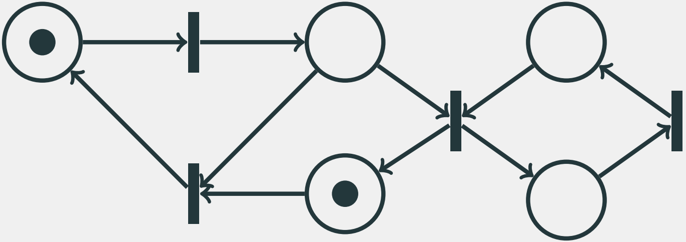


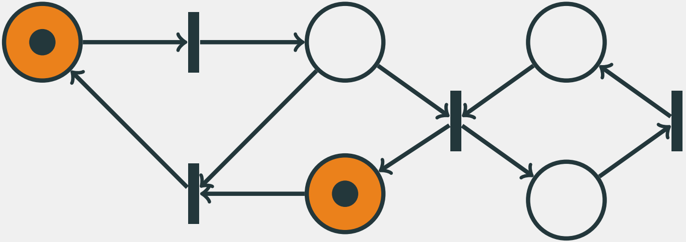
Simulate a "breadth-first" transitions firing



Simulate a "breadth-first" transitions firing
by numbering places/transitions

Verma, Seidl & Schwentick '05

Encoding the firing set conditions



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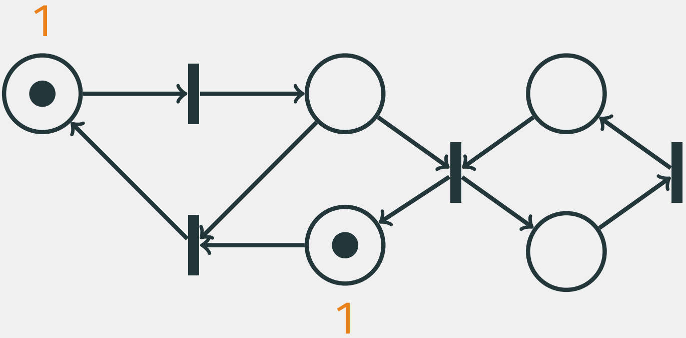

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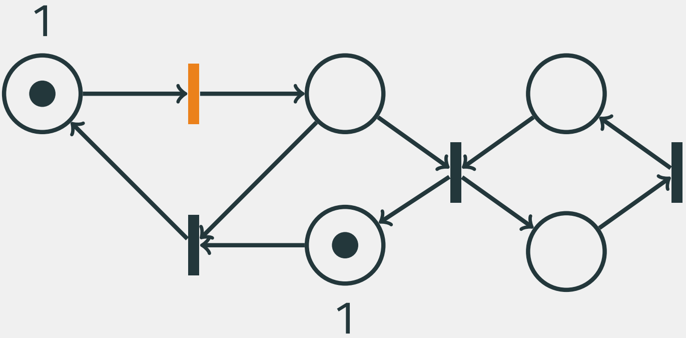

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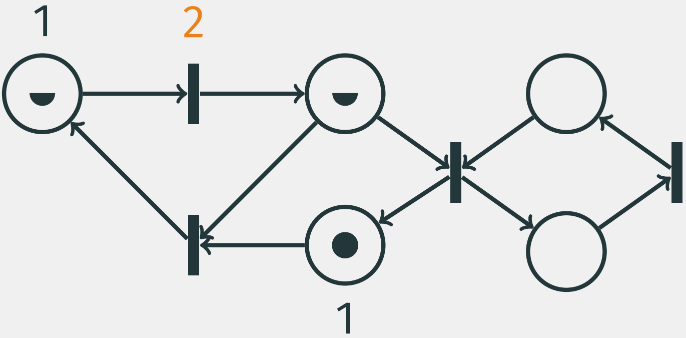

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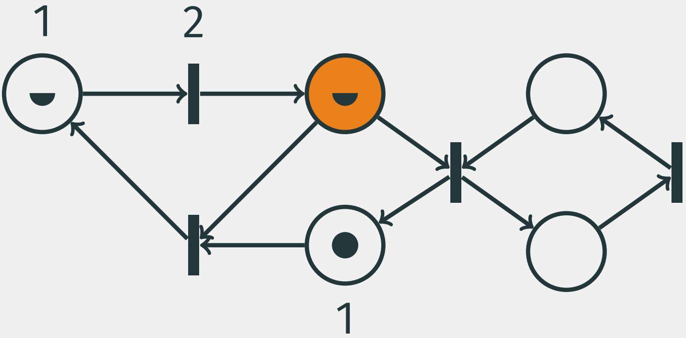

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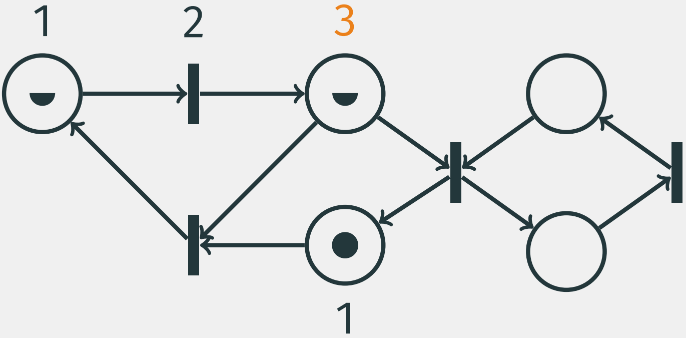

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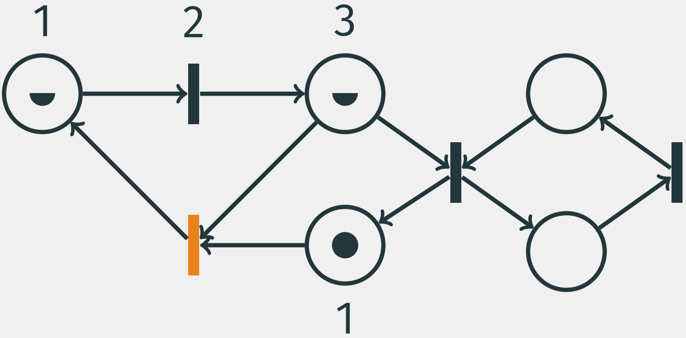

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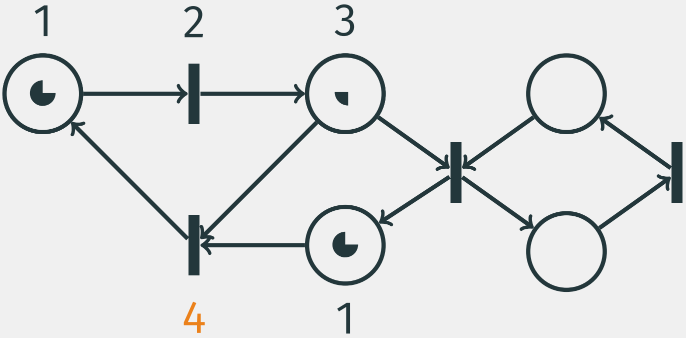

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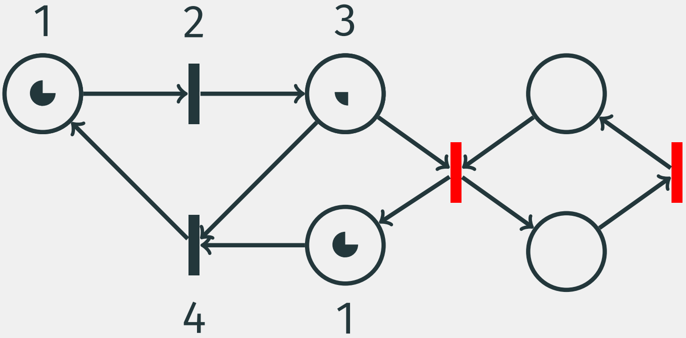

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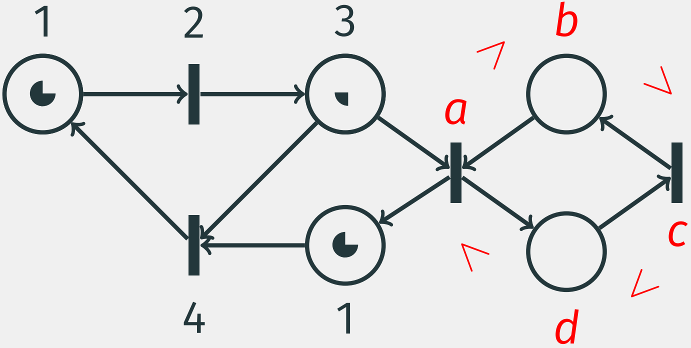

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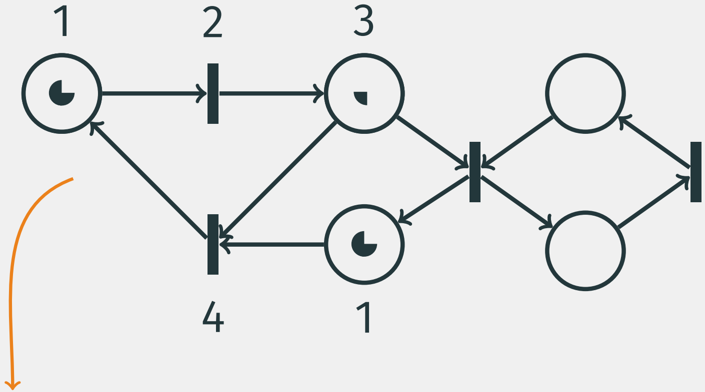
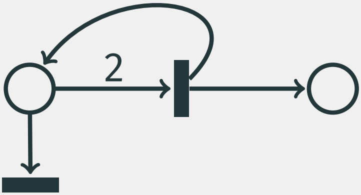
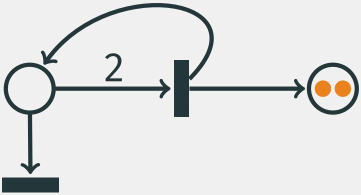


$$\varphi(\boldsymbol{x}) = \exists \boldsymbol{y} : \bigwedge_{p \in P} \boldsymbol{y}(p) > 0 \to \bigwedge_{t \in {}^\bullet p} \boldsymbol{y}(t) < \boldsymbol{y}(p) \cdots$$

From continuous to discrete coverability

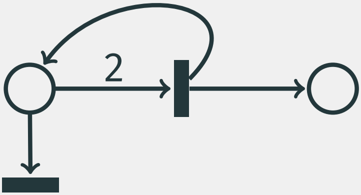


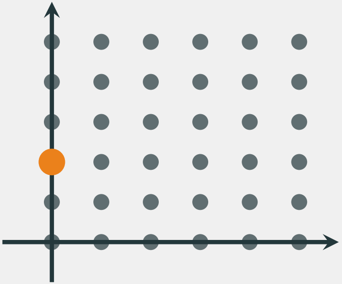
From continuous to discrete coverability



Can $(0, 2)$ be covered from $\boldsymbol{m}_{\text{init}}$?

From continuous to discrete coverability





Backward algorithm

(Arnold & Latteux '78, Abdulla et al. '96)

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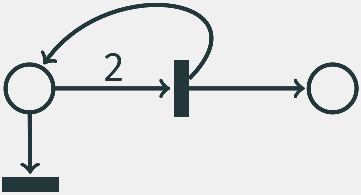

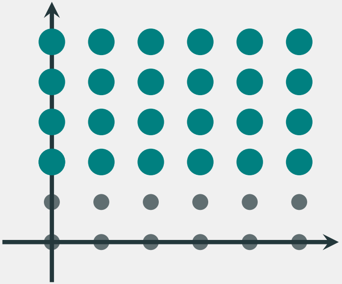

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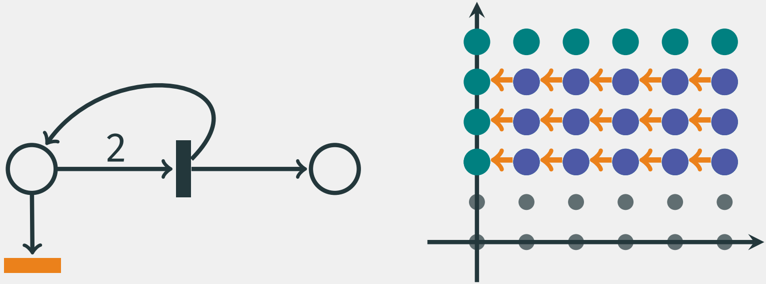

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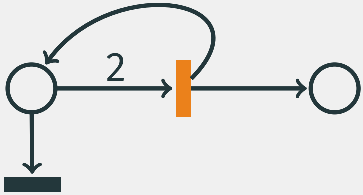

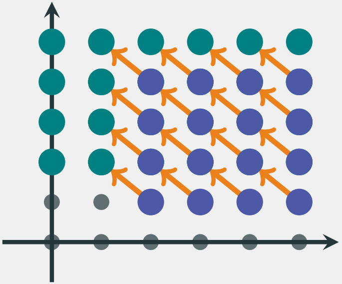

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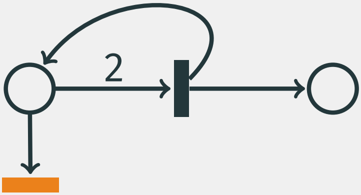

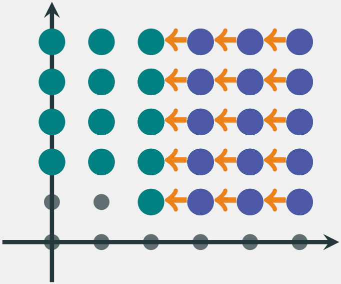

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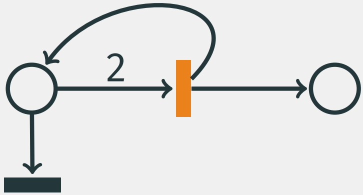

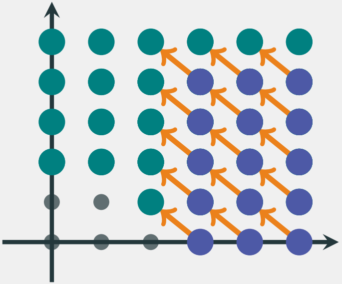

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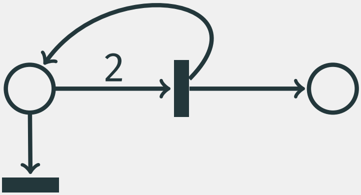

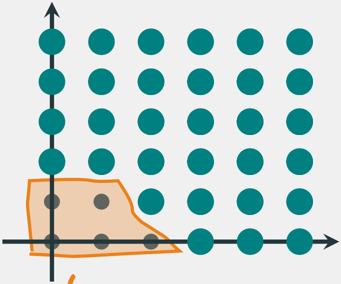

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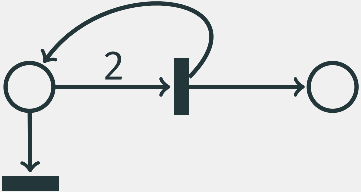
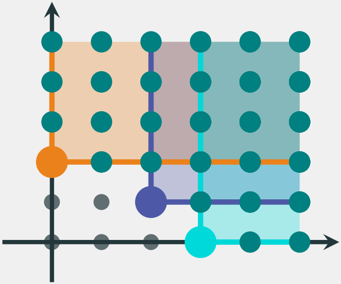
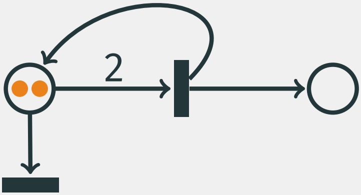
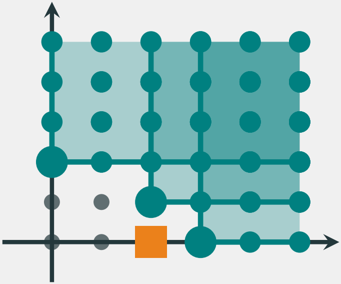




Basis size may become doubly exponential

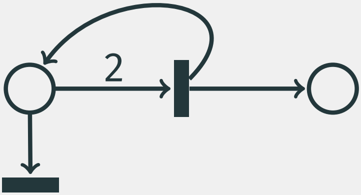
(Bozzelli & Ganty '11)

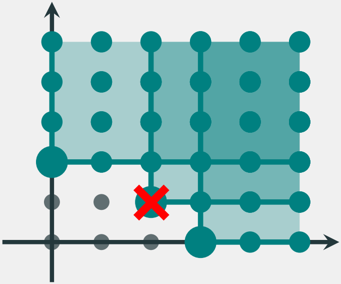




We only care about $\boldsymbol{m}_{\mathrm{init}}$

From continuous to discrete coverability





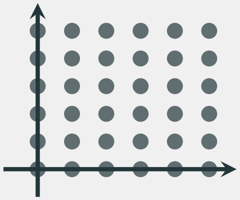
We only care about $\boldsymbol{m}_{\text{init}}$

Prune basis with $\mathbb{Q}$-reachability!

Backward coverability modulo $\mathbb{Q}$-reachability

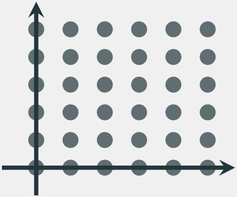
```
if m_target is not ℚ-coverable:
  return false
```

Polynomial time



Backward coverability modulo $\mathbb{Q}$-reachability

```
if m_target is not Q-coverable:
  return false
X = {m_target}
while (m_init not covered by X):
  B = markings obtained from X one step backward
  B = B \ {b ∈ B : ¬φ(b)}
  if B = ∅: return false
  φ(x) = φ(x) ∧ ⋀_{pruned b} x ≱ b
  X = X ∪ B
return true
```



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  φ(x) = φ(x) ∧ ⋀_{pruned b} x ≱ b
  X = X ∪ B
return true
```

if $\boldsymbol{m}_{\text{target}}$ is not $\mathbb{Q}$-coverable:

  return false

$X = \{\boldsymbol{m}_{\text{target}}\}$

while ($\boldsymbol{m}_{\text{init}}$ not covered by X):

  $B =$ markings obtained from X one step backward

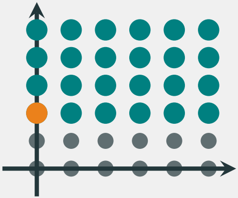
  $B = B \setminus \{\boldsymbol{b} \in B : \neg\varphi(\boldsymbol{b})\}$

  if $B = \emptyset$: return false

  $\varphi(\boldsymbol{x}) = \varphi(\boldsymbol{x}) \ \wedge \ \bigwedge_{\text{pruned } \boldsymbol{b}} \boldsymbol{x} \not\geq \boldsymbol{b}$

  $X = X \cup B$

return true



Backward coverability modulo $\mathbb{Q}$-reachability

`if` $\boldsymbol{m}_{\text{target}}$ is not $\mathbb{Q}$-coverable:

  `return false`

$X = \{\boldsymbol{m}_{\text{target}}\}$

`while` ($\boldsymbol{m}_{\text{init}}$ not covered by X):

  $B =$ markings obtained from X one step backward

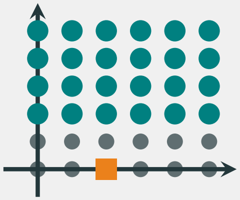
  $B = B \setminus \{\boldsymbol{b} \in B : \neg\varphi(\boldsymbol{b})\}$

  `if` $B = \emptyset$: `return false`

  $\varphi(\boldsymbol{x}) = \varphi(\boldsymbol{x}) \;\wedge\; \bigwedge_{\text{pruned } \boldsymbol{b}} \boldsymbol{x} \not\geq \boldsymbol{b}$

  $X = X \cup B$

`return true`



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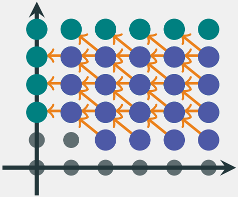

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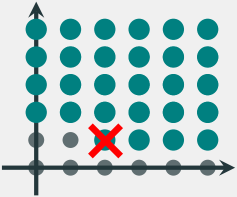
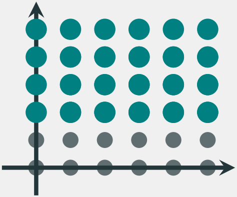
$\mathbb{Q}$-coverability pruning
(better than poly. time)



Backward coverability modulo $\mathbb{Q}$-reachability

```
if m_target is not Q-coverable:
    return false
X = {m_target}
while (m_init not covered by X):
    B = markings obtained from X one step backward
    B = B \ {b ∈ B : ¬φ(b)}
    if B = ∅: return false
    φ(x) = φ(x) ∧ ⋀_{pruned b} x ≱ b
    X = X ∪ B
return true
```



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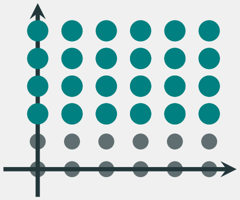
SMT solver guidance



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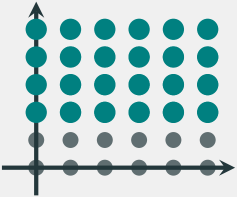

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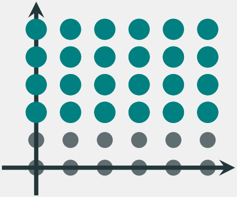


Our implementation : QCover

B., Finkel, Haase & Haddad '16

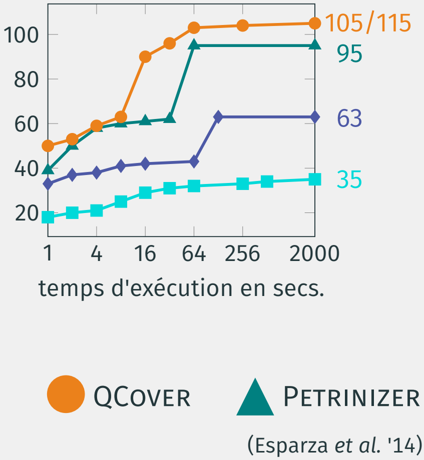
python™ + SMT Solver Z3 (Microsoft Research)

`https://github.com/blondimi/qcover`

Benchmarked on...

- 176 Petri nets (avg. 1054 places, 8458 trans.)
- multi-threaded C programs with shared-memory
- `Erlang` concurrent programs
- Protocols : mutual exclusion, communication, etc.
- Messages provenance analysis : medical and bug-tracking sys.

Instances proven safe



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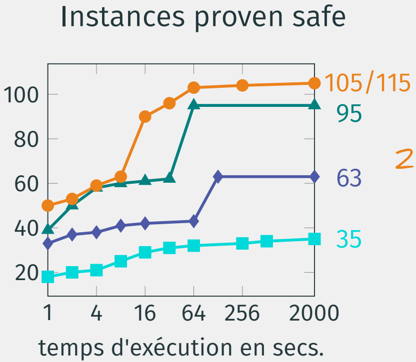

Largest nets proven safe:

21143 places
7150 trans.
42 secs.

6690 places
11934 trans.
21 secs.

754 places
27370 trans.
3 secs.

QCover | Petrinizer (Esparza *et al.* '14) | BFC (Kaiser *et al.* '12) | MIST (Ganty *et al.* '07)

Our implementation : QCover

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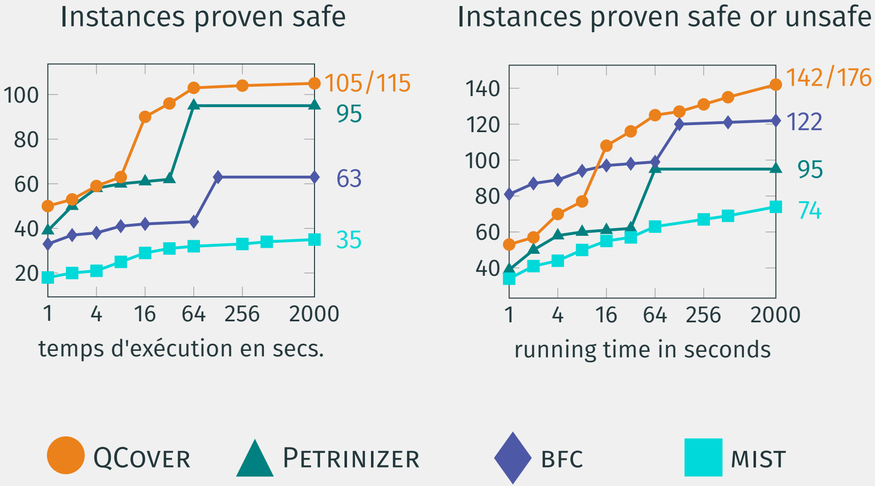
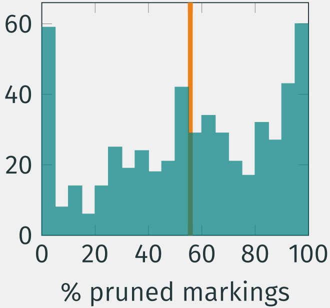
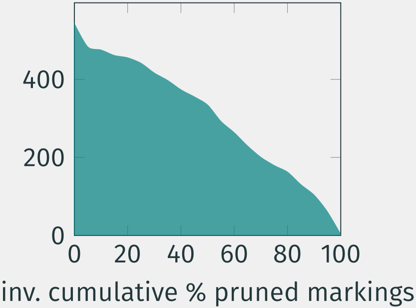


(Esparza *et al.* '14) (Kaiser *et al.* '12) (Ganty *et al.* '07)

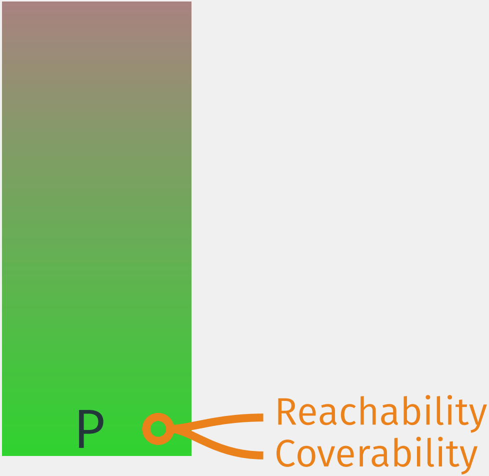
Markings pruning efficiency across all iterations





Another application : from logic to complexity

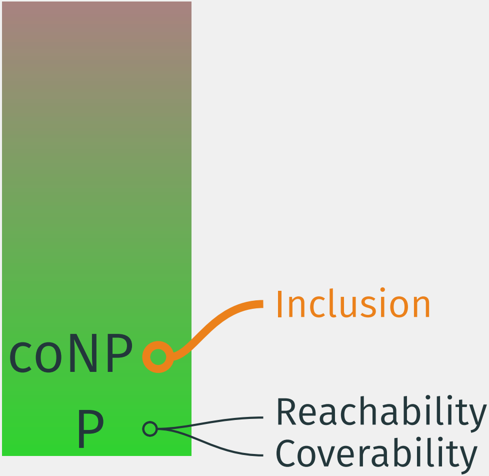
Continuous



(Fraca & Haddad '13)

Another application : from logic to complexity

Continuous



Continuous

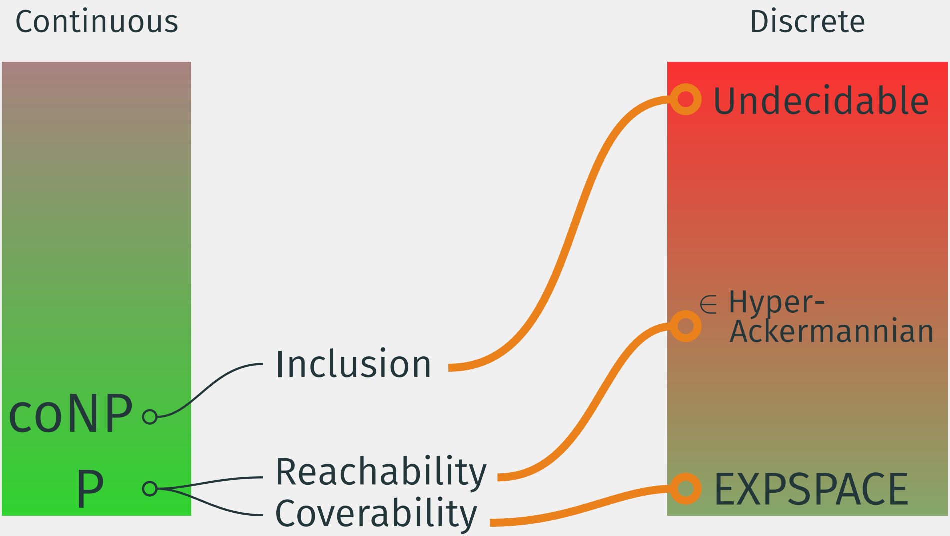
$$\forall \boldsymbol{m}\ \varphi(\boldsymbol{m}_{\text{init}}, \boldsymbol{m}) \rightarrow \varphi'(\boldsymbol{m}'_{\text{init}}, \boldsymbol{m})$$

+ bounds on sys. linear inequalities

coNP — Inclusion

P — Reachability, Coverability

Another application : from logic to complexity



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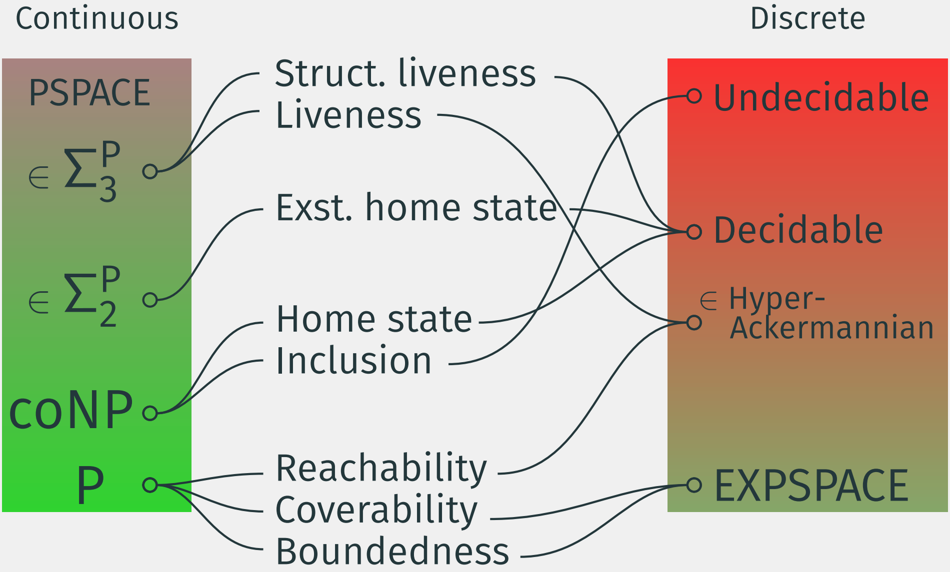

Future work

- Support Petri net extensions : transfers/resets
- Combine our approach with a forward algorithm
- Use upward closed sets data stuctures
  (e.g. sharing trees Delzanno *et al.* '04)
- Continuous vector addition systems with states (VASS)

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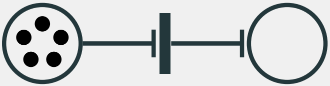

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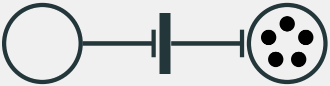

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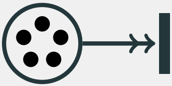

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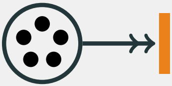

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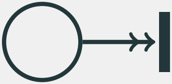

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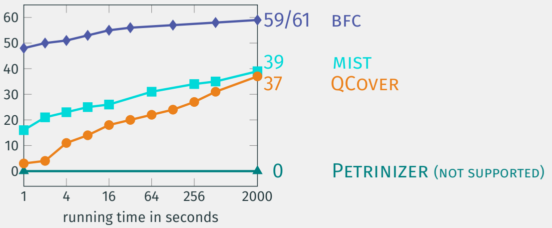

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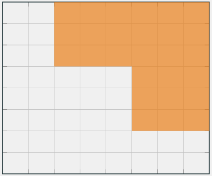

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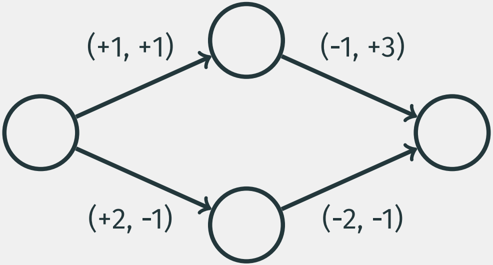

Thank you !

Vielen Dank !