On Tools for Coverability

Michael Blondin, Christoph Haase, Grégoire Sutre

Part I: QCover

Michael Blondin

Joint work with Alain Finkel, Christoph Haase and Serge Haddad

Verifying safety with Petri nets

Process 1

Process 2

Lamport mutual exclusion "1-bit algorithm"

Verifying safety with Petri nets

Process 1

critical section

Process 2

critical section

Lamport mutual exclusion "1-bit algorithm"

Verifying safety with Petri nets

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while True:
  x = True
  while y: pass
  # critical section
  x = False
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  while True:
★   y = True
    if x then:
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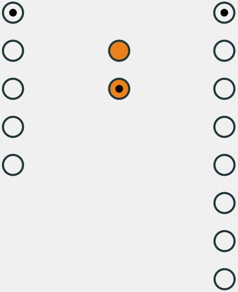

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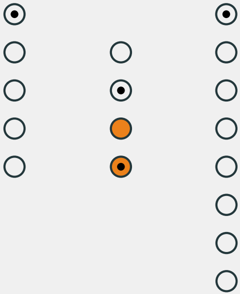

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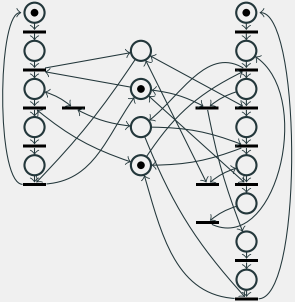

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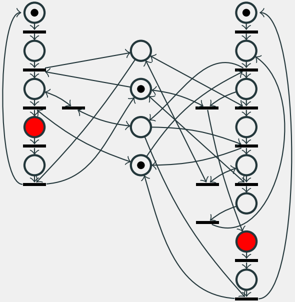

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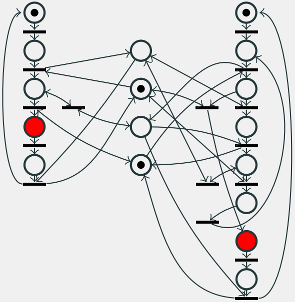
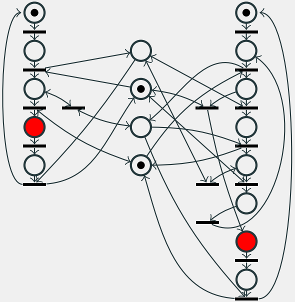


Processes at both critical sections $\iff$ each  ≥ 1

Verifying safety with Petri nets



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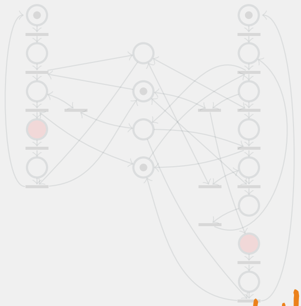

Coverability problem

Processes at both critical sections $\Longleftrightarrow$ each ● ≥ 1 ○ ≥ 0

Coverability problem

Problem

Input:	Petri net $\mathcal{N}$, initial marking $\boldsymbol{m}_0$, target marking $\boldsymbol{m}$
Question:	Is some $\boldsymbol{m}' \geq \boldsymbol{m}$ reachable from $\boldsymbol{m}_0$ in $\mathcal{N}$?

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How to solve it?

- Forward: build reachability tree from initial marking
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- EXPSPACE-complete

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How to solve it? **Karp & Miller '69**

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How to solve it? **Arnold & Latteux '78, Abdulla *et al.* '96**

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How to solve it? **Lipton '76, Rackoff '78**

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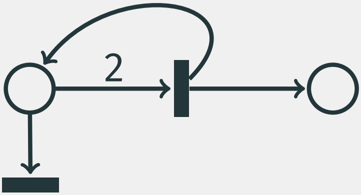

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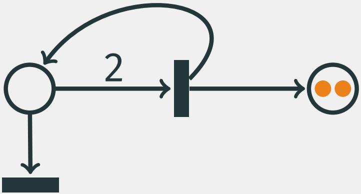
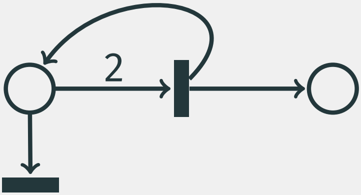
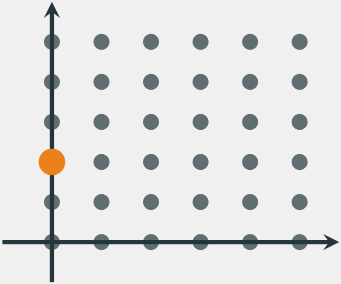


What initial markings may cover $(0, 2)$?

Backward algorithm





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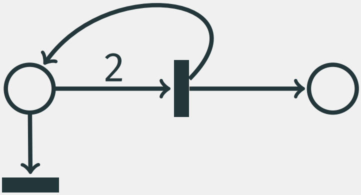

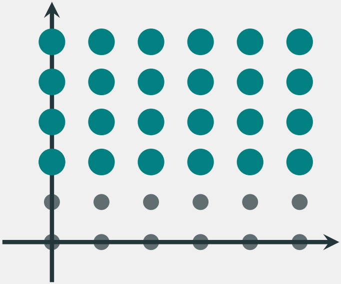

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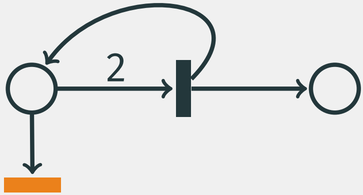

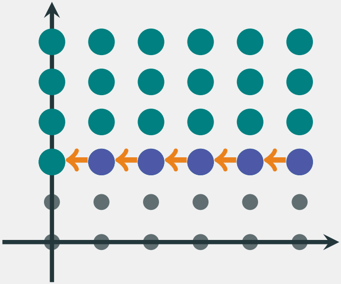

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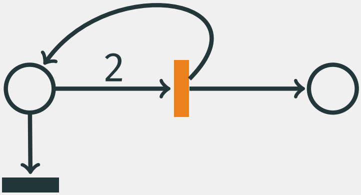

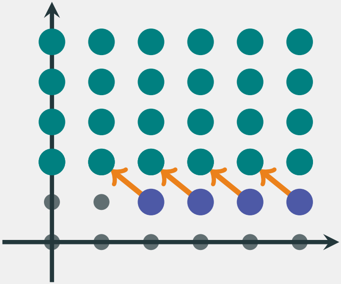

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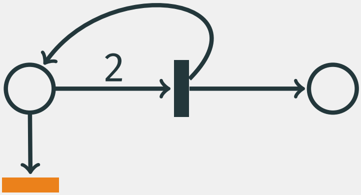

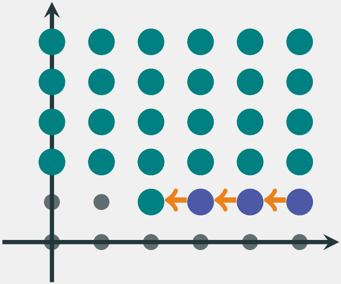

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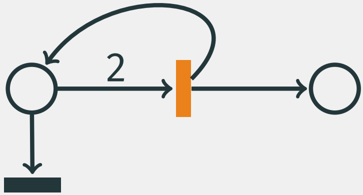

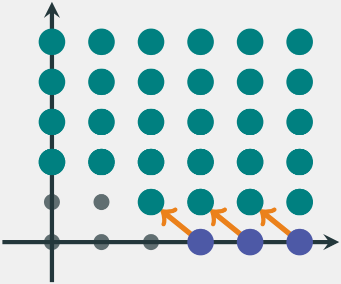

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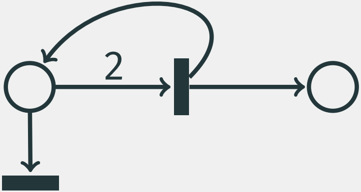

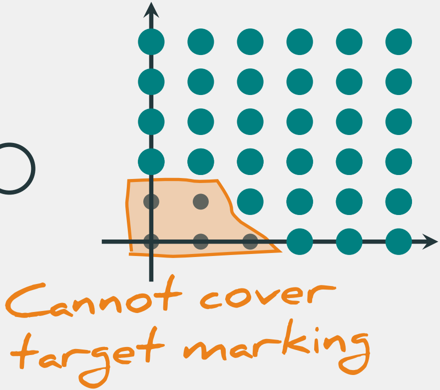

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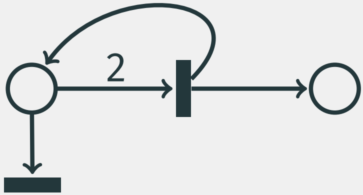
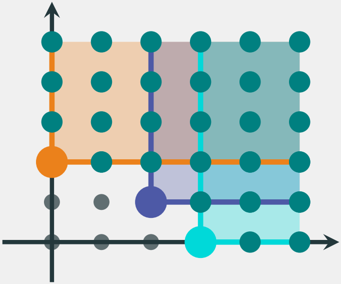
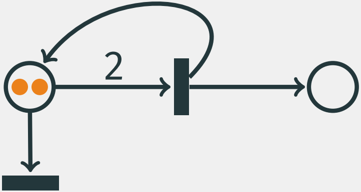
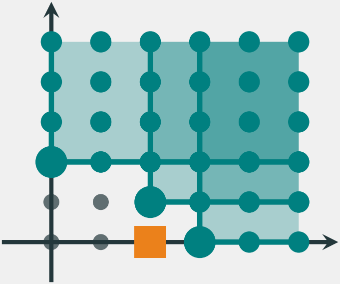




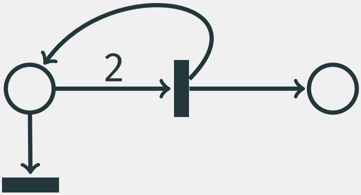
Basis size may become doubly exponential

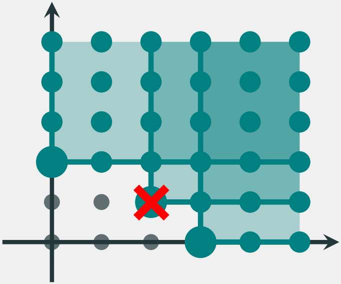
(Bozzelli & Ganty '11)





We only care about some initial marking...





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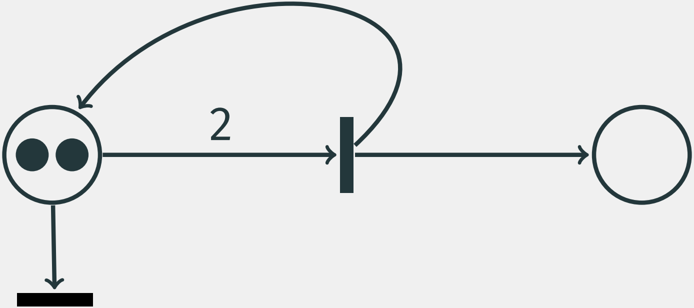
Speedup by pruning basis!

(Discrete) Petri nets



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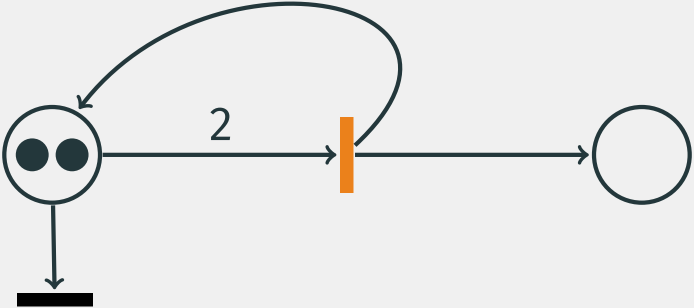

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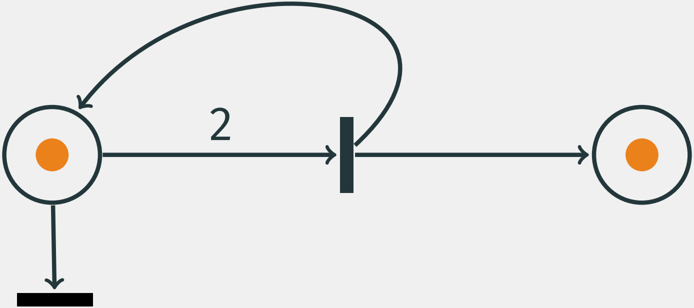

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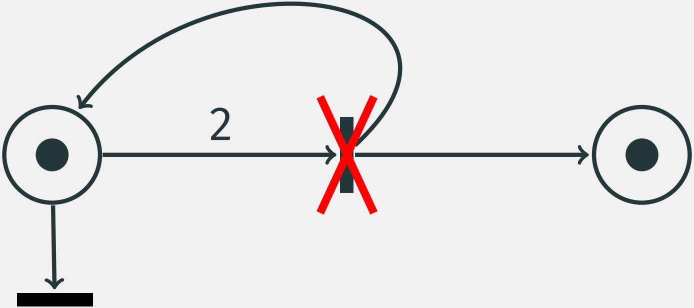

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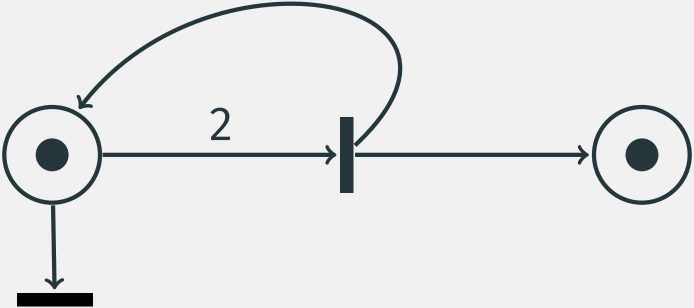

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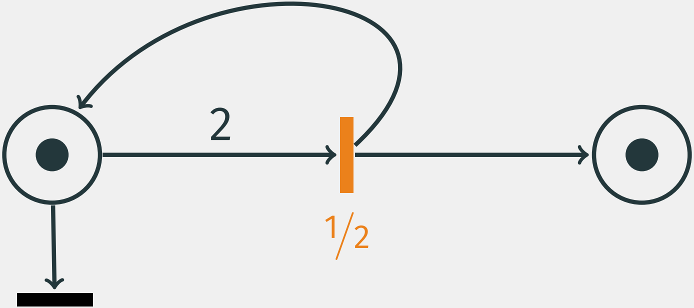

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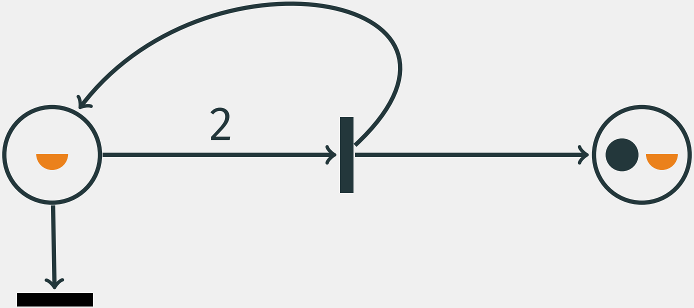

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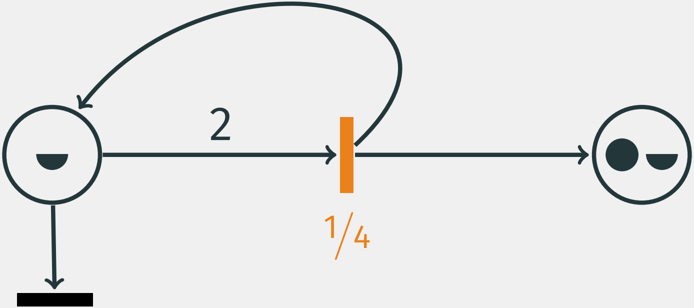

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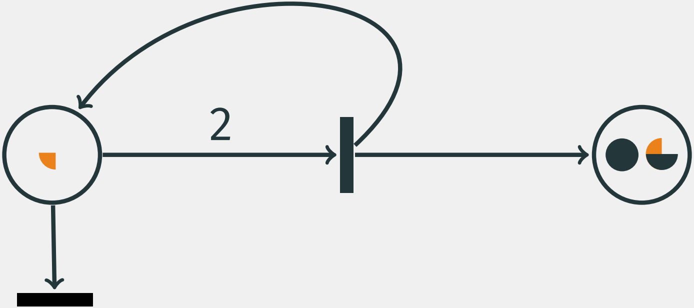

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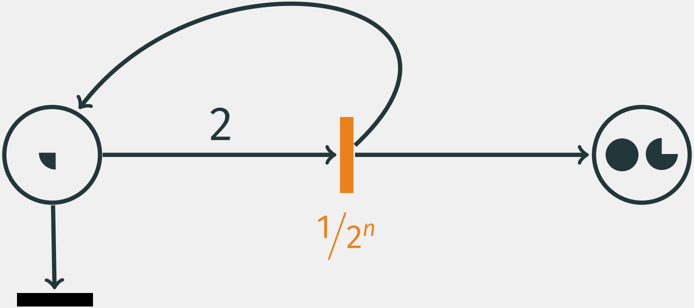

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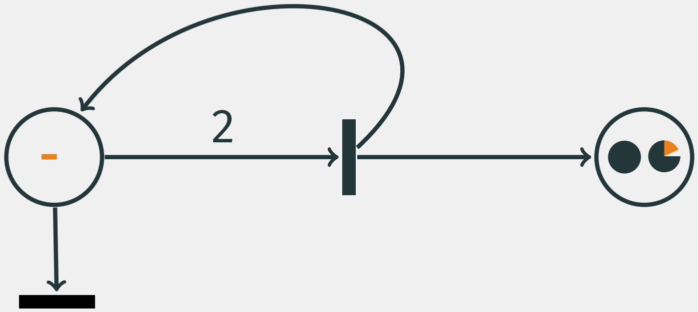

Continuity to over-approximate coverability

$$\boldsymbol{m} \text{ is coverable from } \boldsymbol{m}_0$$

$$\Downarrow$$

$$\boldsymbol{m} \text{ is } \mathbb{Q}\text{-coverable from } \boldsymbol{m}_0$$

$\boldsymbol{m}$ is coverable from $\boldsymbol{m}_0$

EXPSPACE

$$\Downarrow$$

$\boldsymbol{m}$ is $\mathbb{Q}$-coverable from $\boldsymbol{m}_0$

PTIME

$$\Downarrow \quad \not\Uparrow$$

$\boldsymbol{m}_0$ and $\boldsymbol{m}$ satisfy conditions of
Esparza, Ledesma-Garza, Majumdar, Meyer & Niksic '14

NP / EXPTIME

Continuity to over-approximate coverability

$\boldsymbol{m}$ is not coverable from $\boldsymbol{m}_0$

Safety

$$\Uparrow$$

$\boldsymbol{m}$ is not $\mathbb{Q}$-coverable from $\boldsymbol{m}_0$

Coverability in continuous Petri nets

Fix some continuous Petri net $(P, T, \mathbf{Pre}, \mathbf{Post})$

$\boldsymbol{m}$ is $\mathbb{Q}$-coverable from $\boldsymbol{m}_0$ iff... **Fraca & Haddad '13**

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there exist $\boldsymbol{m}' \geq \boldsymbol{m}$ and $\boldsymbol{v} \in \mathbb{Q}_{\geq 0}^{T}$ such that

- $\boldsymbol{m}' = \boldsymbol{m_0} + (\mathbf{Post} - \mathbf{Pre}) \cdot \boldsymbol{v}$

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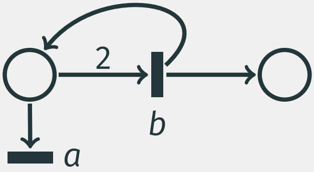

$$\boldsymbol{m}_0 = (2, 0)$$
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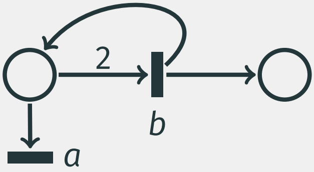

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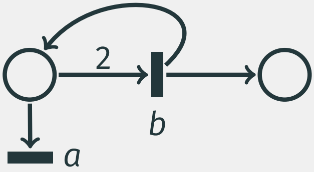

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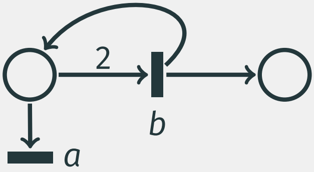

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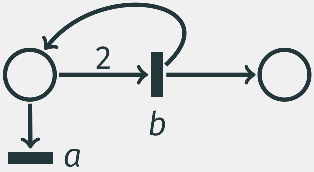

$$\boldsymbol{m}_0 = (2, 0)$$
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$\boldsymbol{m}$ is $\mathbb{Q}$-coverable from $\boldsymbol{m}_0$ iff... **Fraca & Haddad '13**

there exist $\boldsymbol{m}' \geq \boldsymbol{m}$ and $\boldsymbol{v}_a, \boldsymbol{v}_b \in \mathbb{Q}_{\geq 0}$ such that

- $\begin{array}{l} 0 \leq \boldsymbol{v}_b + \boldsymbol{v}_a \leq 2 \\ 2 \leq \boldsymbol{v}_b \end{array} \implies \boldsymbol{v}_a = 0,\ \boldsymbol{v}_b = 2,\ \boldsymbol{m}' = \boldsymbol{m}$ ✓
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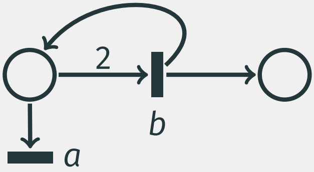
Coverability in continuous Petri nets



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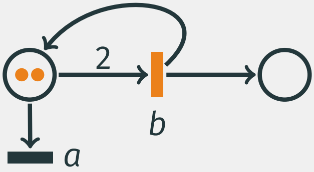
Coverability in continuous Petri nets



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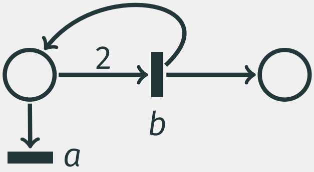
Coverability in continuous Petri nets



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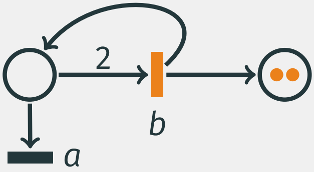

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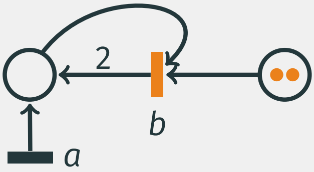
Coverability in continuous Petri nets



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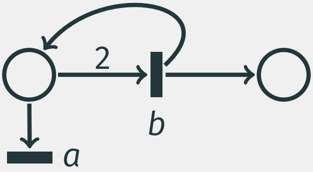
Coverability in continuous Petri nets



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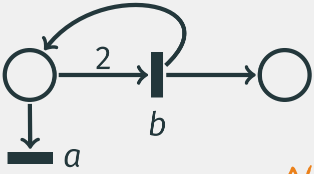
- some execution from $\boldsymbol{m}_0$ fires exactly $\{b\}$ ✓
- some execution to $\boldsymbol{m}'$ fires exactly $\{b\}$ ✗

Coverability in continuous Petri nets



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Not $\mathbb{Q}$-coverable from

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- some execution from $\boldsymbol{m}_0$ fires exactly $\{b\}$ ✓
- some execution to $\boldsymbol{m}'$ fires exactly $\{b\}$ ✗

Coverability in continuous Petri nets

Polynomial time!

$\boldsymbol{m}$ is $\mathbb{Q}$-coverable from $\boldsymbol{m}_0$ iff... **Fraca & Haddad '13**

there exist $\boldsymbol{m}' \geq \boldsymbol{m}$ and $\boldsymbol{v} \in \mathbb{Q}_{\geq 0}^{T}$ such that

- $\boldsymbol{m}' = \boldsymbol{m_0} + (\mathbf{Post} - \mathbf{Pre}) \cdot \boldsymbol{v}$
- some execution from $\boldsymbol{m}_0$ fires exactly $\{t \in T : \boldsymbol{v}_t > 0\}$
- some execution to $\boldsymbol{m}'$ fires exactly $\{t \in T : \boldsymbol{v}_t > 0\}$

Coverability in continuous Petri nets

Logical characterization **TACAS'16**

$\mathbb{Q}$-coverability can be encoded in a linear size formula of

existential $\text{FO}(\mathbb{Q}_{\geq 0}, +, <)$

$\boldsymbol{m}$ is $\mathbb{Q}$-coverable from $\boldsymbol{m}_0$ iff... **Fraca & Haddad '13**

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Coverability in continuous Petri nets

Logical characterization TACAS'16

$\mathbb{Q}$-coverability can be encoded in a linear size formula of

$$\text{existential FO}(\mathbb{N}, \quad +, <)$$

Even better approximation

$\boldsymbol{m}$ is $\mathbb{Q}$-coverable from $\boldsymbol{m}_0$ iff... Fraca & Haddad '13

there exist $\boldsymbol{m}' \geq \boldsymbol{m}$ and $\boldsymbol{v} \in \mathbb{Q}^T_{\geq 0}$ such that

- $\boldsymbol{m}' = \boldsymbol{m}_0 + (\mathbf{Post} - \mathbf{Pre}) \cdot \boldsymbol{v}$
- some execution from $\boldsymbol{m}_0$ fires exactly $\{t \in T : \boldsymbol{v}_t > 0\}$
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Logical characterization TACAS'16

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m **is $\mathbb{Q}$-coverable from $\boldsymbol{m}_0$ iff...** Fraca & Haddad '13

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- $\boldsymbol{m}' = \boldsymbol{m_0} + (\textbf{Post} - \textbf{Pre}) \cdot \boldsymbol{v}$ Straightforward
- some execution from $\boldsymbol{m}_0$ fires exactly $\{t \in T : \boldsymbol{v}_t > 0\}$
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Coverability in continuous Petri nets

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- $\boldsymbol{m}' = \boldsymbol{m_0} + (\mathbf{Post} - \mathbf{Pre}) \cdot \boldsymbol{v}$

More subtle

- some execution from $\boldsymbol{m}_0$ fires exactly $\{t \in T : \boldsymbol{v}_t > 0\}$
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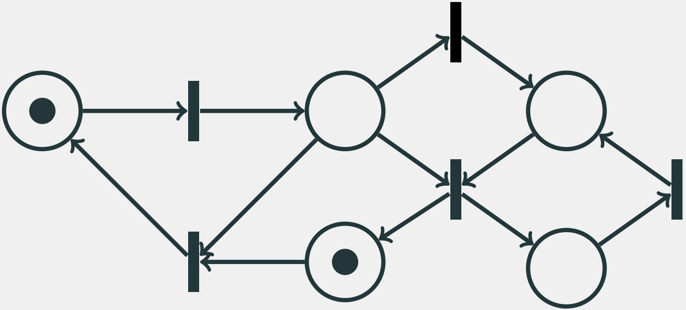
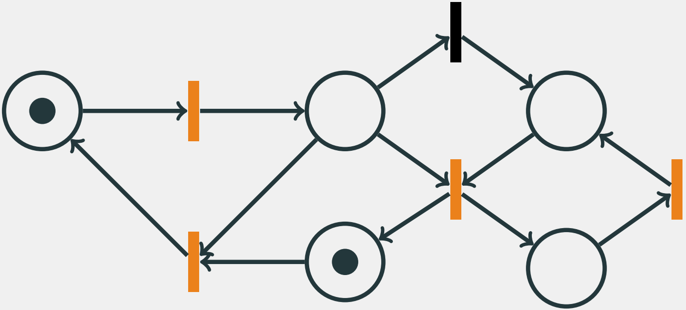
Encoding the firing set conditions



Testing whether some transitions can be fired from initial marking

Encoding the firing set conditions



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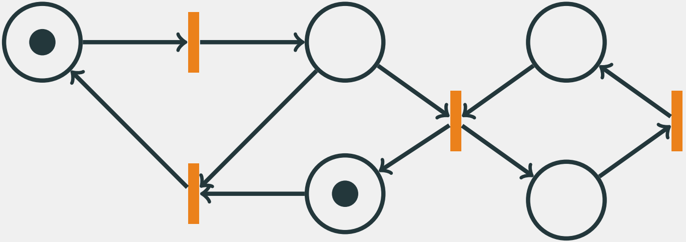

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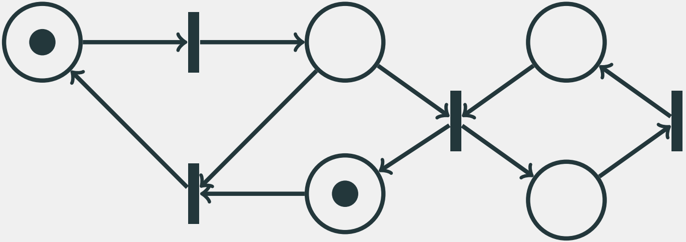
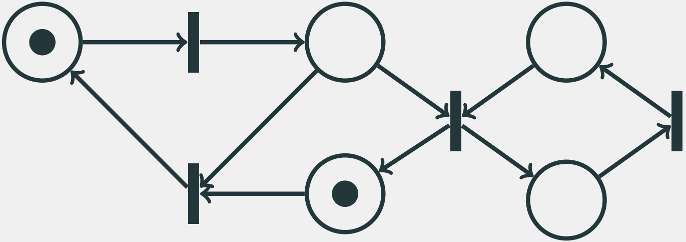


Simulate a "breadth-first" transitions firing

Encoding the firing set conditions

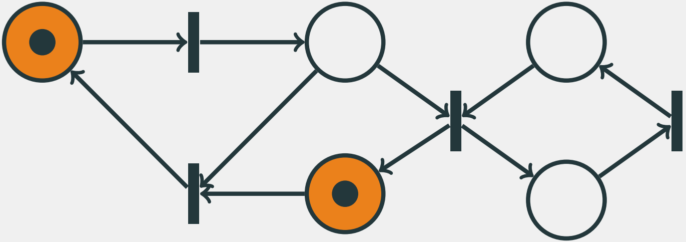


Simulate a "breadth-first" transitions firing

by numbering places/transitions

(Verma, Seidl & Schwentick '05)

Encoding the firing set conditions



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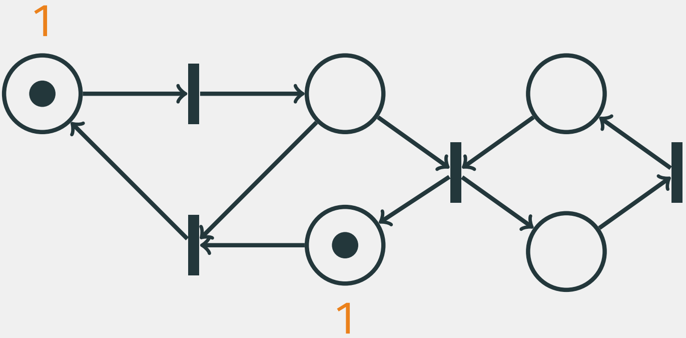

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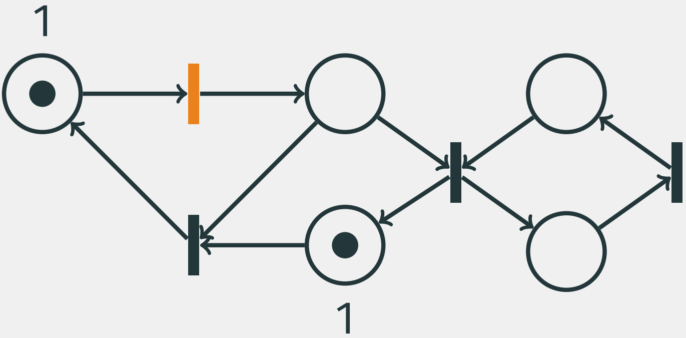

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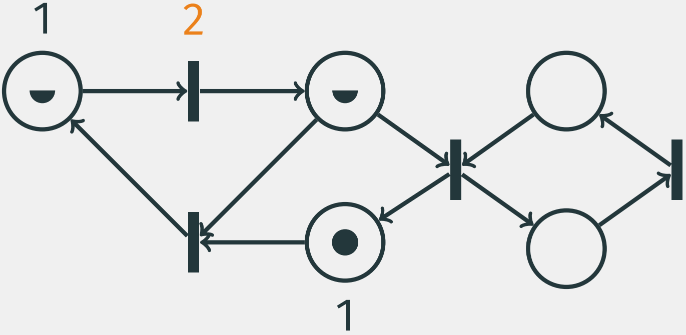

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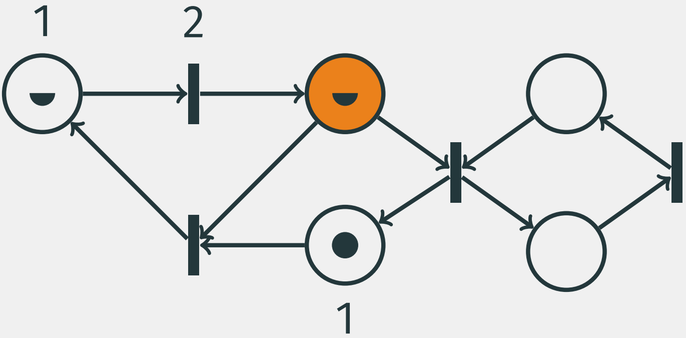

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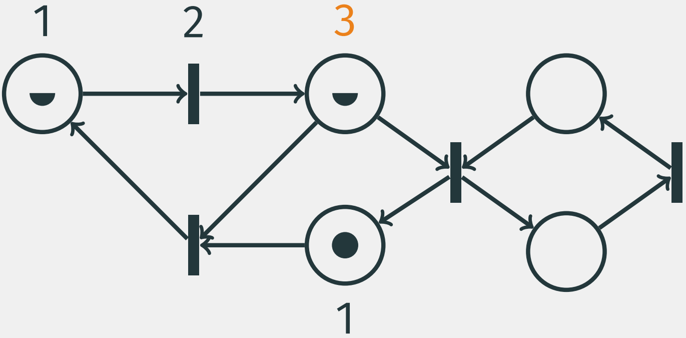

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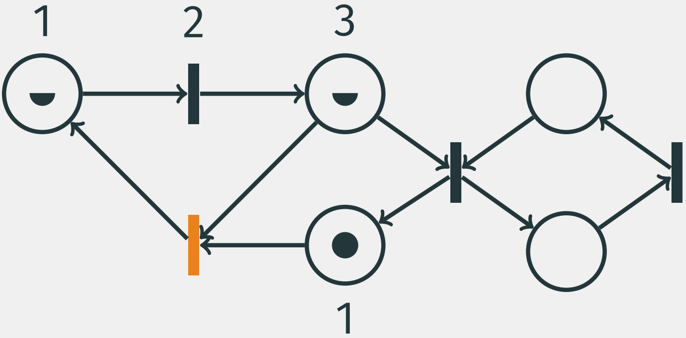

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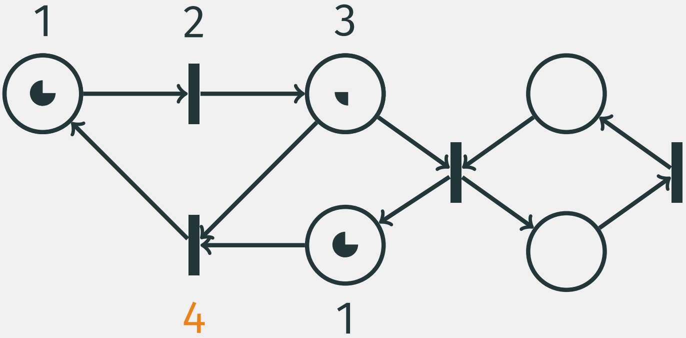

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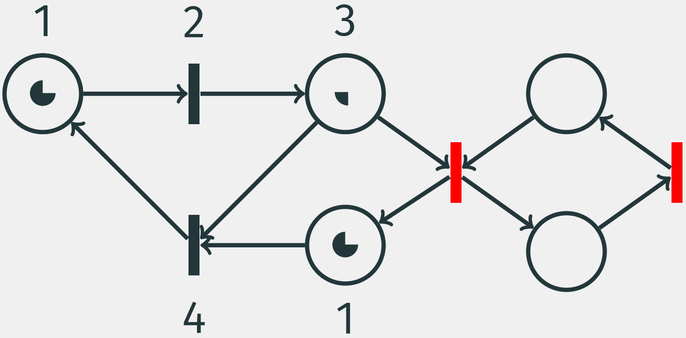

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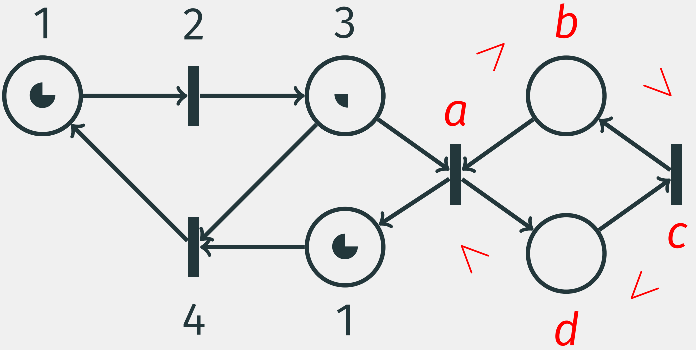

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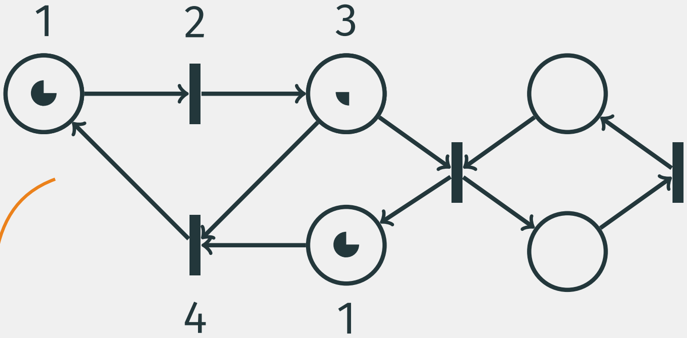

$$\varphi(\boldsymbol{x}) = \exists \boldsymbol{y} : \bigwedge_{p \in P} \boldsymbol{y}(p) > 0 \rightarrow \bigwedge_{t \in {}^{\bullet}p} \boldsymbol{y}(t) < \boldsymbol{y}(p) \cdots$$

Backward coverability modulo $\mathbb{Q}$-coverability

```
if target marking m is not ℚ-coverable:
  return False
```

Polynomial time

Backward coverability modulo $\mathbb{Q}$-coverability

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if target marking m is not ℚ-coverable:
  return False
X = {target marking m}
while (initial marking m₀ not covered by X):
  B = markings obtained from X one step backward
  B = B \ {b ∈ B : ¬φ(b)}
  if B = ∅: return False
  φ(x) = φ(x) ∧ ⋀_{pruned b} x ≱ b
  X = X ∪ B
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`while` (initial marking $\boldsymbol{m}_0$ not covered by X):

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  $B = B \setminus \{\boldsymbol{b} \in B : \neg\varphi(\boldsymbol{b})\}$

  `if` $B = \emptyset$: `return False`

  $\varphi(\boldsymbol{x}) = \varphi(\boldsymbol{x}) \ \wedge \ \bigwedge_{\text{pruned } \boldsymbol{b}} \boldsymbol{x} \not\geq \boldsymbol{b}$

  $X = X \cup B$

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SMT solver guidance

Backward coverability modulo $\mathbb{Q}$-coverability

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An implementation: QCover

python™ + SMT solver Z3 (Microsoft Research)

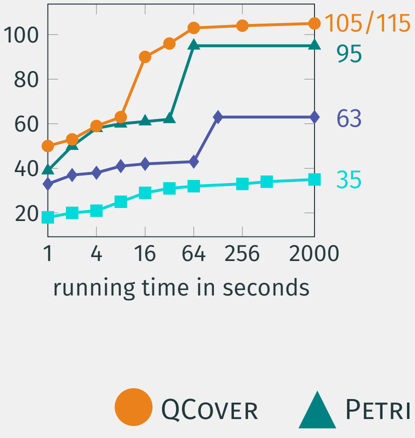
`https://github.com/blondimi/qcover`

Tested on...

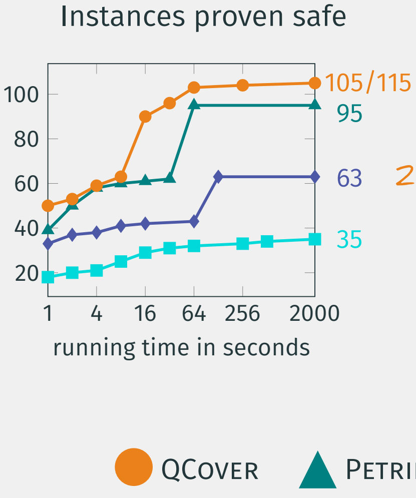
- 176 Petri nets (avg. 1054 places, 8458 transitions)
- C/Erlang programs with threads
- Mutual exclusion protocols, communication protocols, etc.
- Message analysis of a medical and a bug tracking system

An implementation: QCover

Instances proven safe



An implementation: QCover



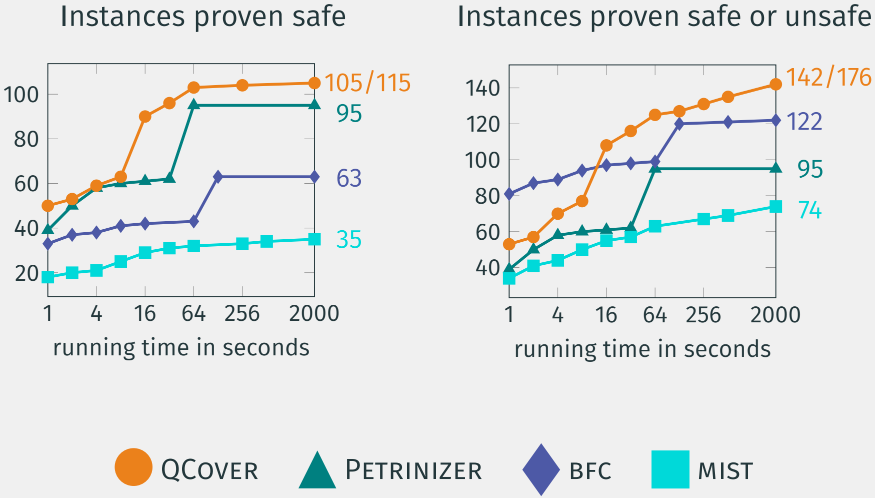
Largest nets proved safe:

21143 places
7150 trans.
42 secs.

6690 places
11934 trans.
21 secs.

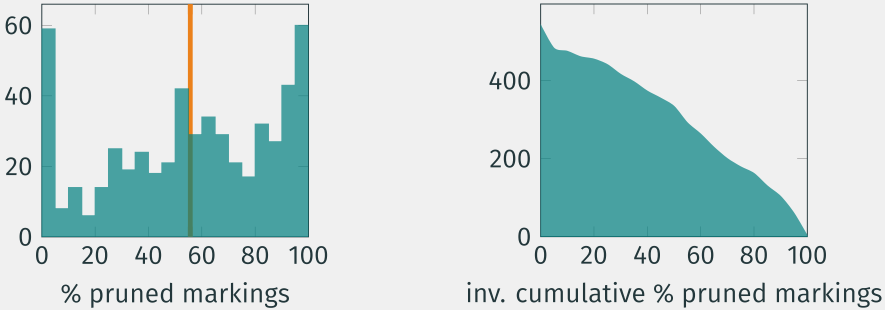
754 places
27370 trans.
3 secs.

An implementation: QCover



An implementation: QCover

Markings pruning efficiency across all iterations



Possible extensions

- Combine our approach with a forward algorithm to better handle unsafe instances

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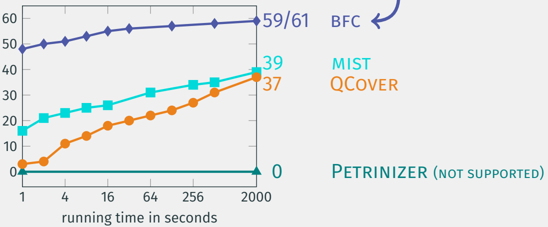


Possible extensions

- Combine our approach with a forward algorithm to better handle unsafe instances
- Use more efficient data structures, *e.g.* sharing trees

(Delzanno, Raskin & Van Begin '04)

Possible extensions

- Combine our approach with a forward algorithm to better handle unsafe instances

- Use more efficient data structures, *e.g.* sharing trees

  (Delzanno, Raskin & Van Begin '04)

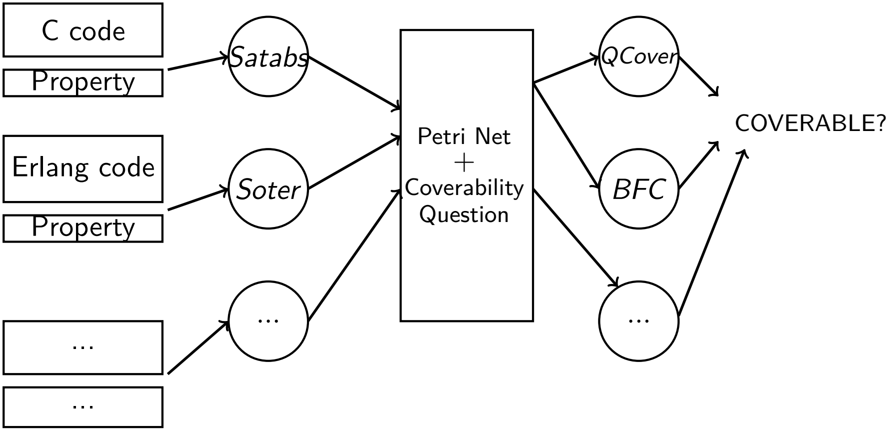
- Support Petri nets extensions

Part II: ICover

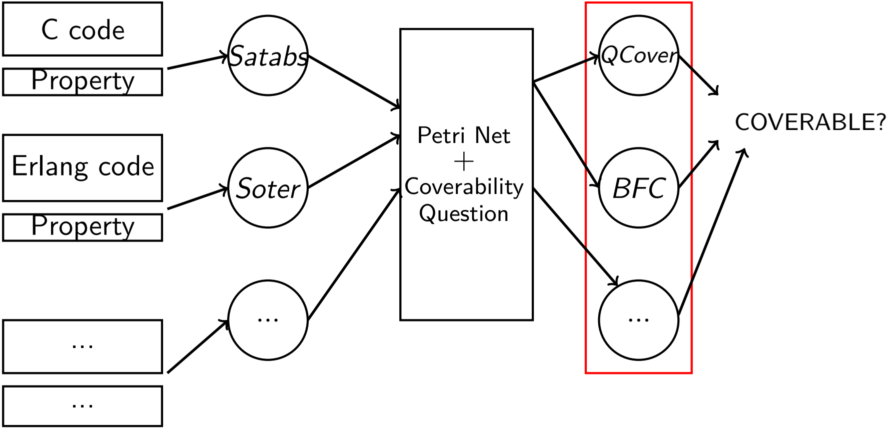
Grégoire Sutre

Joint work with Thomas Geffroy and Jérôme Leroux

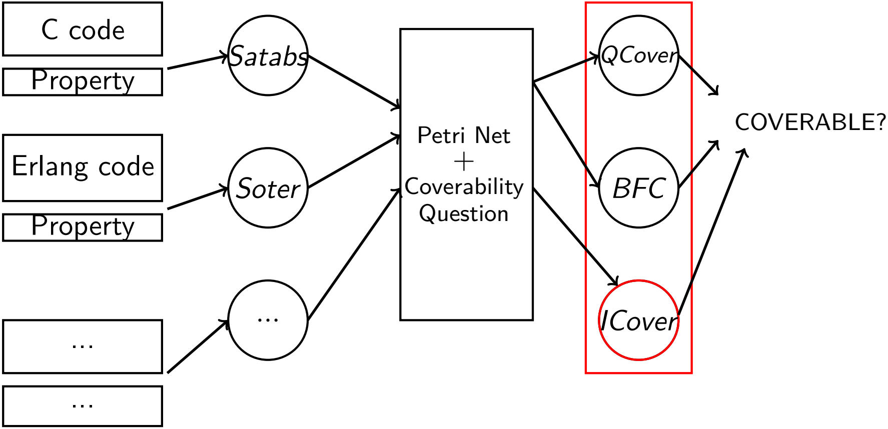
Verifying Systems with Petri Nets



Verifying Systems with Petri Nets



Verifying Systems with Petri Nets



Coverability in Petri nets

$$\mathit{init} \xrightarrow{*} m \geq \mathit{target}?$$

Decidability - Complexity

- Decidable (Karp and Miller - 1969)
- $\textsc{ExpSpace}$-complete (Lipton - 1976, Rackoff - 1978)

Coverability in Petri nets

$$init \xrightarrow{*} m \geq target?$$

Tools

Mist (Ganty, Geeraerts, Raskin, Van Begin, . . .)

- interval sharing trees
- backward search + place invariants
- abstraction refinement

BFC (Kaiser, Kroening, Wahl)

Target set widening + forward Karp-Miller

Petrinizer (Esparza, Ledesma-Garza, Majumdar, Meyer, Niksic)

SMT, state equation + traps

QCover (Blondin, Finkel, Haase, Haddad)

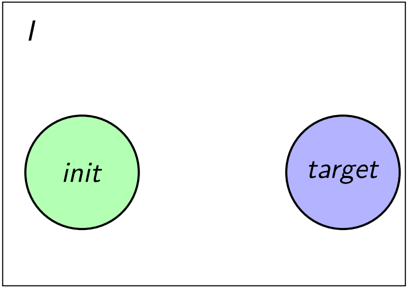
SMT, continuous reachability + backward search

ICover: Generalisation of *QCover* with Invariants

Assumption:

1. I is an invariant (I contains all reachable markings)
2. I is a downward closed set

$U_0 := \uparrow(target \cap I)$



ICover: Generalisation of *QCover* with Invariants

Assumption:

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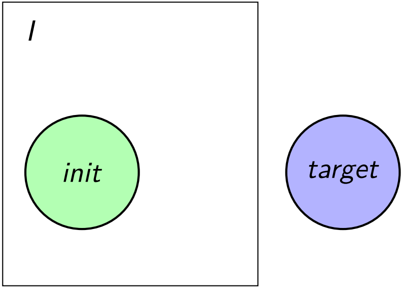
$U_0 := \emptyset$: Safe !



ICover: Generalisation of *QCover* with Invariants

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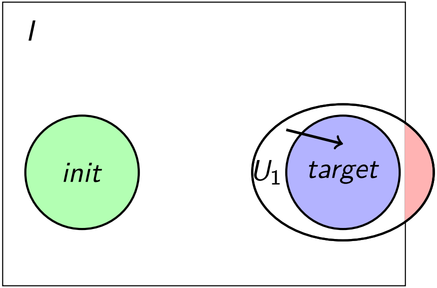
$U_0 := \uparrow(target \cap I)$

$U_1 := U_0 \cup \uparrow(pre(U_0) \cap I)$



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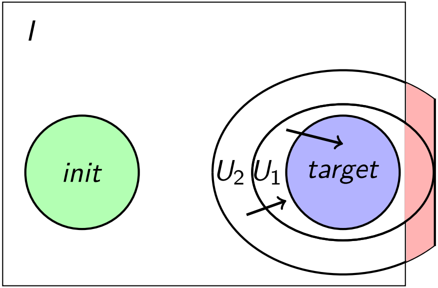


$U_0 := \uparrow(target \cap I)$

$U_1 := U_0 \cup \uparrow(pre(U_0) \cap I)$
$U_2 := U_1 \cup \uparrow(pre(U_1) \cap I)$

ICover: Generalisation of *QCover* with Invariants

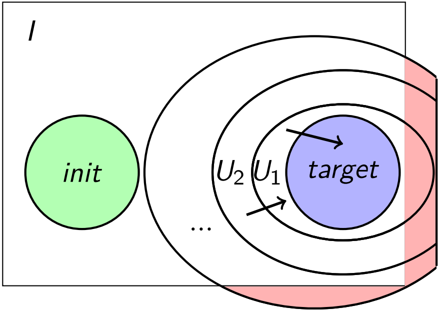
Assumption:

1. I is an invariant (I contains all reachable markings)
2. I is a downward closed set

$U_0 := \uparrow(\mathit{target} \cap I)$

$U_1 := U_0 \cup \uparrow(\mathit{pre}(U_0) \cap I)$
$U_2 := U_1 \cup \uparrow(\mathit{pre}(U_1) \cap I)$
$\ldots$
$U_{k+1} := U_k \cup \uparrow(\mathit{pre}(U_k) \cap I)$

Always terminates
(Dickson's lemma)



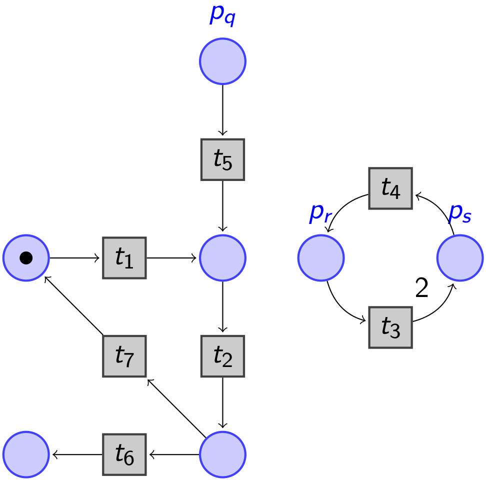
$U_{n+1} = U_n$

Backward Algorithm with Invariant-Based Pruning

if $target \in I$ **then**
  $B \leftarrow \{target\}$;
else
  return `False`;
end
while $m_{init} \notin \uparrow B$ **do**
  $N \leftarrow min(pre(\uparrow B)) \setminus \uparrow B$
  $P \leftarrow N \cap I$
  if $P = \emptyset$ **then**
    return `False`;
  end
  $B \leftarrow \min(B \cup P)$;
end
return `True`;

- I is an invariant
- I is a downward closed set

Invariant: Sign Analysis



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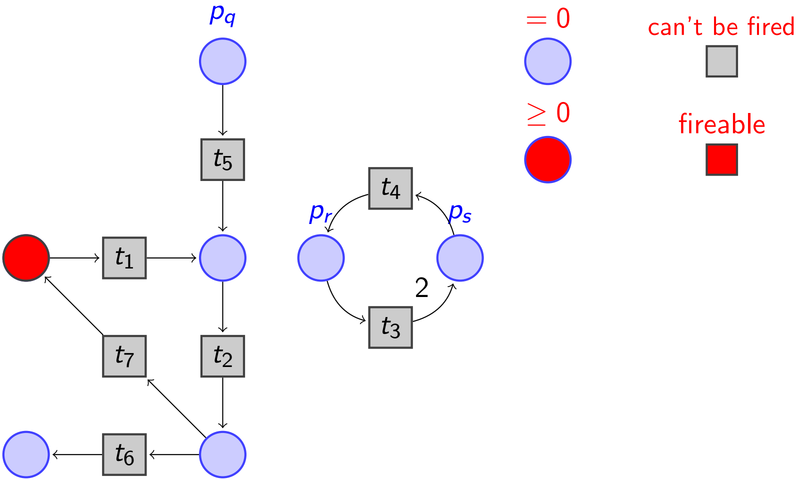

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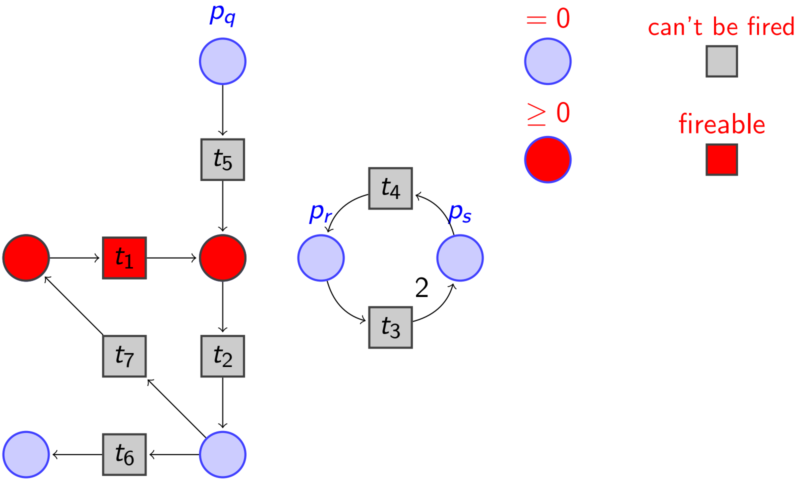
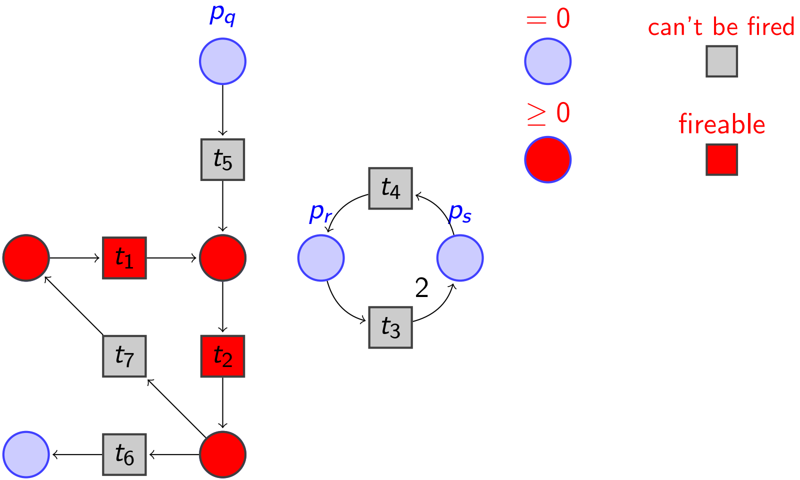
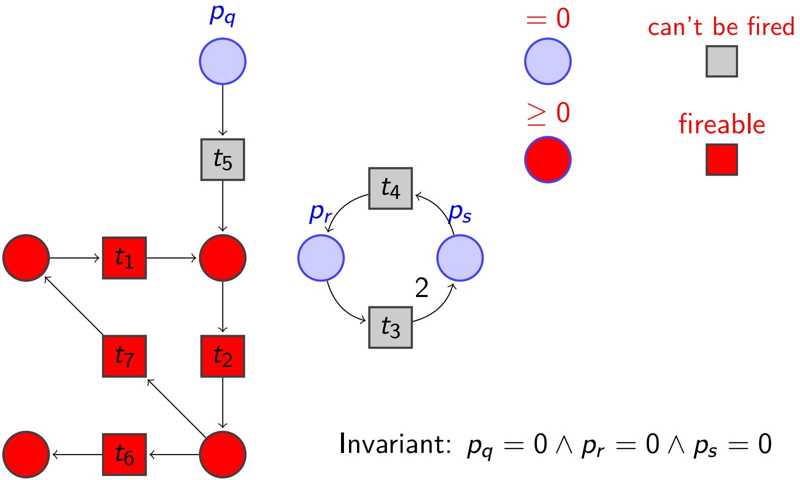


Invariant: Sign Analysis

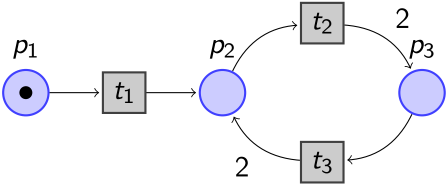


Invariant: Sign Analysis



Invariant: $p_q = 0 \wedge p_r = 0 \wedge p_s = 0$

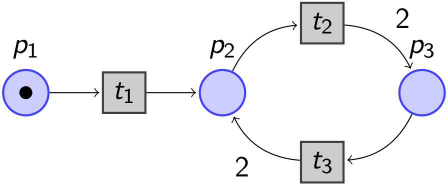
Invariant: State Inequation



$$p_1 \xrightarrow{t_1} \xrightarrow{t_2} \xrightarrow{t_3} \dots \xrightarrow{t_2} \xrightarrow{t_3} r$$

$$\Delta(t_1) = p_2 - p_1$$
$$\Delta(t_2) = 2p_3 - p_2$$
$$\Delta(t_3) = 2p_2 - p_3$$

Invariant: State Inequation



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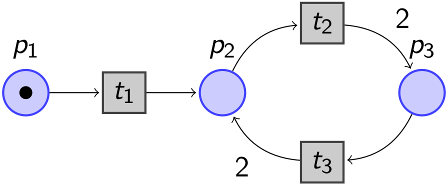
$$\Delta(t_3) = 2p_2 - p_3$$

x_i: number of occurrences of t_i

$$r = init + x_1\Delta(t_1) + x_2\Delta(t_2) + x_3\Delta(t_3)$$

Invariant: State Inequation



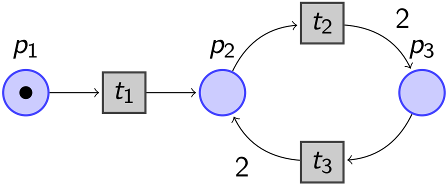
$$p_1 \xrightarrow{t_1}\xrightarrow{t_2}\xrightarrow{t_3} \dots \xrightarrow{t_2}\xrightarrow{t_3} r \geq m$$

$$\Delta(t_1) = p_2 - p_1$$
$$\Delta(t_2) = 2p_3 - p_2$$
$$\Delta(t_3) = 2p_2 - p_3$$

x_i: number of occurrences of t_i

$$m \leq r = init + x_1\Delta(t_1) + x_2\Delta(t_2) + x_3\Delta(t_3)$$

Invariant: State Inequation



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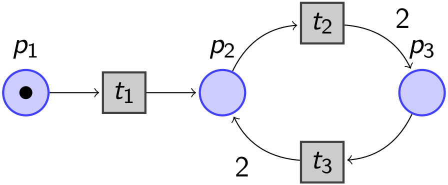
x_i: number of occurrences of t_i

$$m \leq r = init + x_1\Delta(t_1) + x_2\Delta(t_2) + x_3\Delta(t_3)$$

Invariant

$I := \{m \mid \exists x, init + \sum_{t \in T} x(t)\Delta(t) \geq m\}$

Invariant: State Inequation



$$p_1 \xrightarrow{t_1} \xrightarrow{t_2} \xrightarrow{t_3} \ldots \xrightarrow{t_2} \xrightarrow{t_3} r \geq m$$

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x_i: number of occurrences of t_i

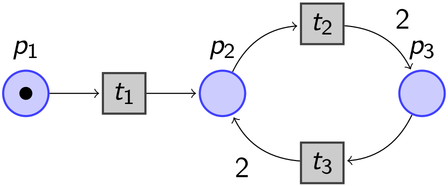
$$m \leq r = init + x_1\Delta(t_1) + x_2\Delta(t_2) + x_3\Delta(t_3)$$

$$\begin{cases} x_1, x_2, x_3 \geq 0 \\ m(p_1) \leq 1 - x_1 \\ m(p_2) \leq x_1 - x_2 + 2x_3 \\ m(p_3) \leq 2x_2 - x_3 \end{cases}$$

Invariant

$I := \{m \mid \exists x, init + \sum_{t \in T} x(t)\Delta(t) \geq m\}$

Invariant: State Inequation



$$p_1 \xrightarrow{t_1}\xrightarrow{t_2}\xrightarrow{t_3} \dots \xrightarrow{t_2}\xrightarrow{t_3} r \geq m$$

$$\Delta(t_1) = p_2 - p_1$$
$$\Delta(t_2) = 2p_3 - p_2$$
$$\Delta(t_3) = 2p_2 - p_3$$

x_i: number of occurrences of t_i

$$p_1 \leq 1 \qquad m \leq r = init + x_1\Delta(t_1) + x_2\Delta(t_2) + x_3\Delta(t_3)$$

Invariant

$I := \{m \mid \exists x, init + \sum_{t \in T} x(t)\Delta(t) \geq m\}$

Experimentations

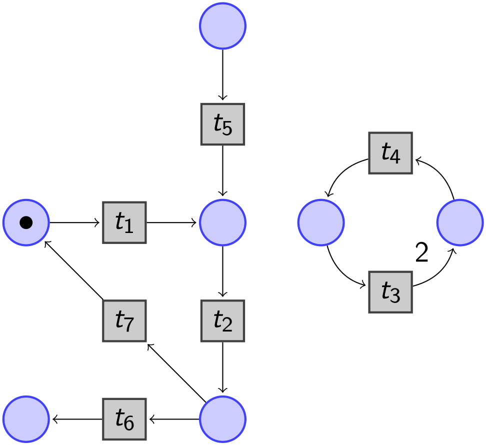
New Tool: *ICover*

- Based on *QCover* written in Python (~900 lines of codes)
- Both use the SMT-Solver *z3* (Bjorner et al. - 2007)
- *ICover* available as a patch of *QCover* (~400 lines of codes)
- `dept-info.labri.u-bordeaux.fr/~tgeffroy/icover`

Results

- Benchmarks (176 instances) used by *QCover* and others
- *QCover* solved 106 / 115 safe instances (2000 seconds per instance)
- *QCover* solved 37 / 61 unsafe instances (idem)
- *ICover* solved as much safe instances and one more unsafe
- It works ! 10 000 seconds (*QCover*) to 5 000 seconds (*ICover*)

Experimentations: Sign Analysis As A Pre-processing



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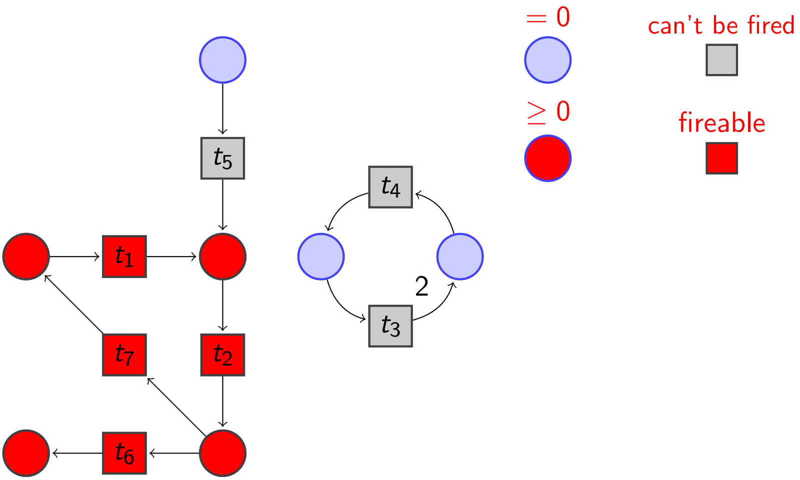

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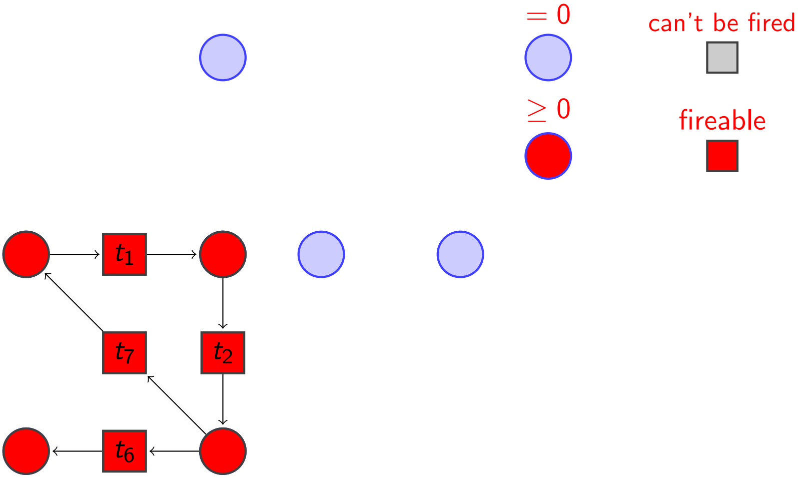
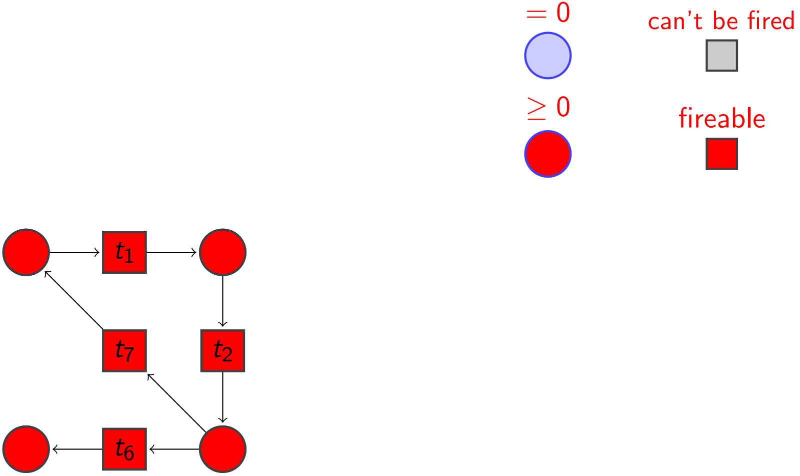
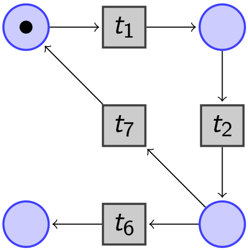


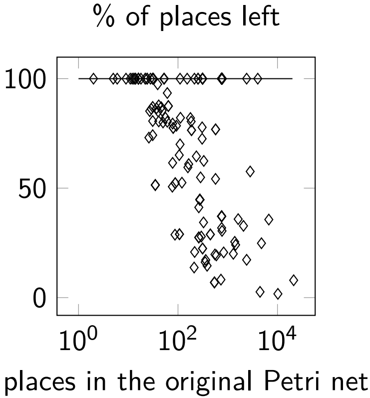
Experimentations: Sign Analysis As A Pre-processing

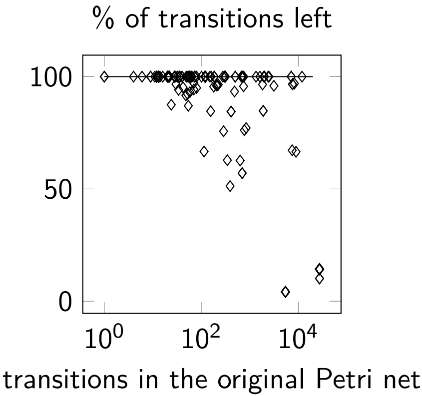


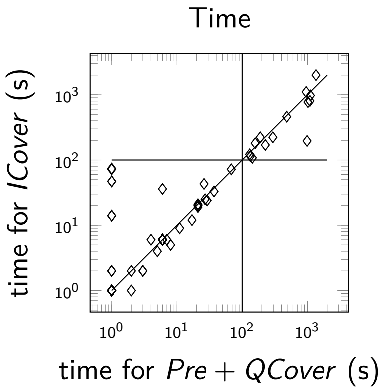
Experimentations: Sign Analysis As A Pre-processing

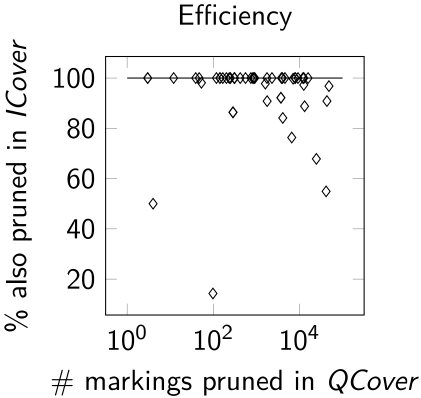


Results of Pre-processing

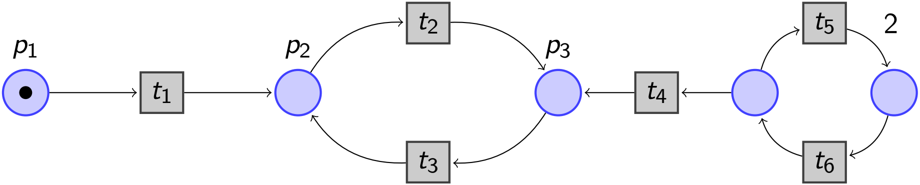





Time
time for ICover (s)
10^0
10^1
10^2
10^3
time for $Pre + QCover$ (s)

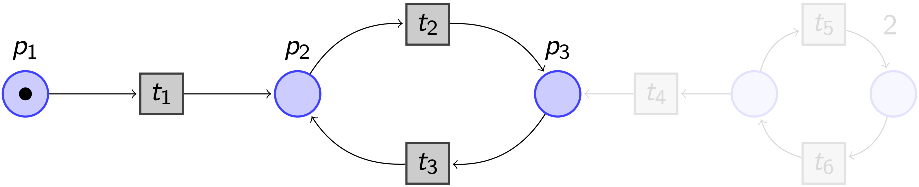

Efficiency
% also pruned in ICover
100
80
60
40
20
10^0
10^2
10^4
markings pruned in QCover

State Inequation More Precise with Pre-Processing



- Can't cover $p_1 + p_2 + p_3$ from p_1
- State inequation: $p_1 \leq 1$ not precise enough
- State inequation: $p_1 + p_2 + p_3 \leq 1$ precise enough

State Inequation More Precise with Pre-Processing



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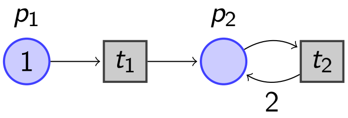
State Inequation vs $\dashrightarrow$

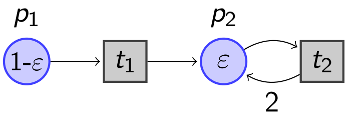


- $p_1 + p_2$ not coverable from p_1 with $\dashrightarrow$
- $p_1 + p_2$ satisfy the state inequation: $p_1 \leq 1$

Theorem (Recalde, Teruel and Silva - 1999)

In a pre-processed Petri net, m satisfies the state inequation iff there exists $m' \geq m$ and a sequence $m_0, m_1, \ldots$ such that init $\overset{*}{\dashrightarrow} m_k$ for every k and such that $m_0, m_1, \ldots$ converges toward m'.

State Inequation vs $\dashrightarrow$



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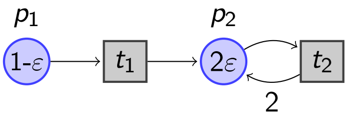

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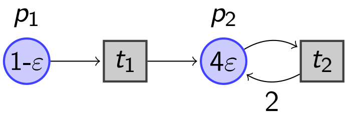

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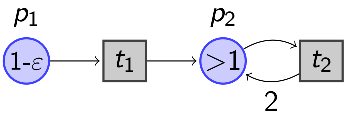
State Inequation vs $\dashrightarrow$



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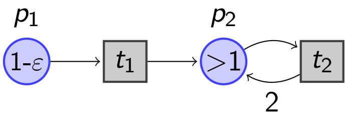

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Conclusion

New

- Backward coverability algorithm with invariant-based pruning
- Pre-processing is a cheap way to accelerate verification
- In practice, in a pre-processed Petri net, state inequation is almost as good as $\dashrightarrow$ coverability

Future work

- Find other cheap pre-processings and invariants
- Apply to other classes of well-structured transition systems

Part III: Best practices

Christoph Haase

Tools...

- increase visibility outside your peer group
- help understanding what is relevant to other people
- generate feedback for theoretical work
- can convince reviewers
- attract students

Before you start

- Choice of language
  - interpreted vs. compiled
  - statically vs. dynamically typed
- Bindings for SMT solver
- Performance of memory operations

Software engineering aspects

- Object oriented programming
- Unit tests
- Documentation
- Use profilers to find bottlenecks

Benchmarks

- One of the most important aspects
- Use other people's benchmarks
- Contact authors if necessary
- Pitfalls:
    - Parsing can entail large costs
    - Avoid unfair treatment of competitors
    - Choose evaluation metrics wisely

Availability

- Obtain institutional clearance $\in \mathbf{F}_\omega$
- Choose license: BSD preferred by industry
- Use public code repositories, e.g. GitHub

Future

- Identify relevant Petri net subclasses and extensions, e.g.
  - business processes
  - process mining
  - population protocols
  - thread transition systems
- Submit to and integrate into existing software competitions

Final words

The SMT solver is always faster than you!

Thank you!   Diolch!