

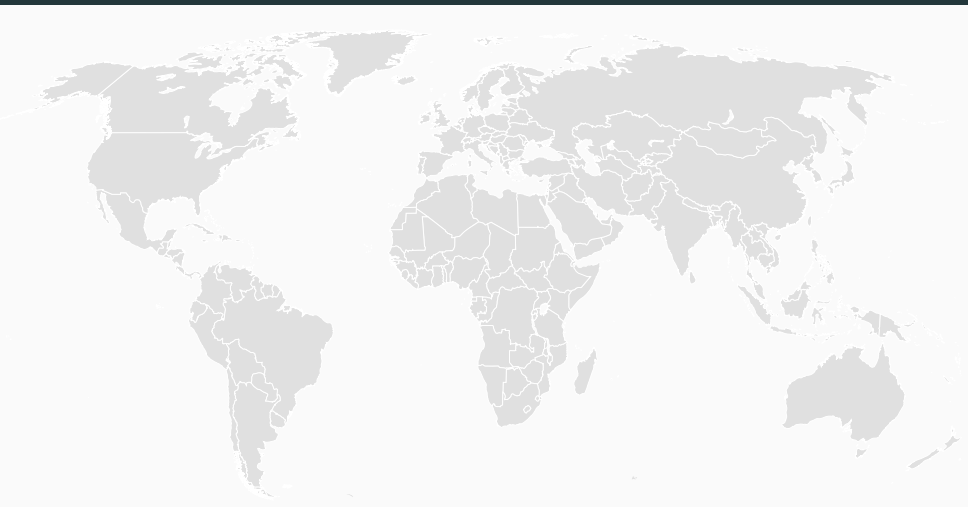
My Humble Journey Through Academia

Michael Blondin



Université de
Sherbrooke

My journey in computer science



My journey in computer science



- DEC @ Collège Lionel-Groulx, 2004 – 2007

My journey in computer science



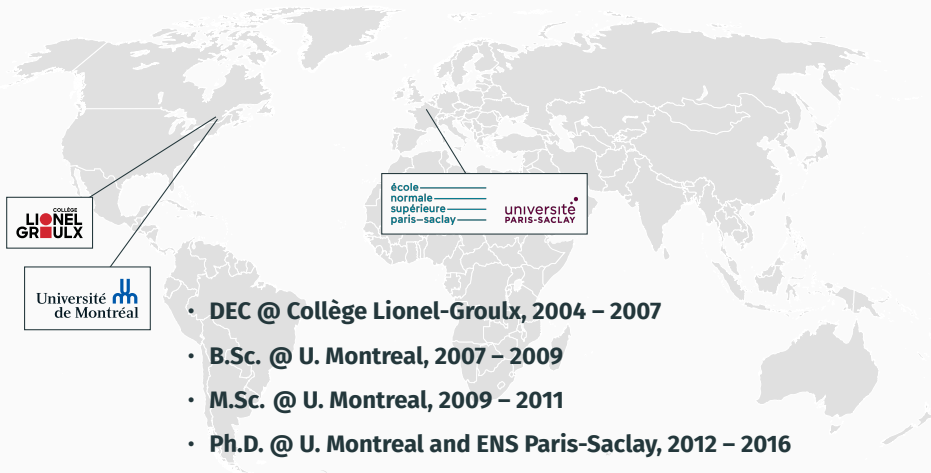
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- Associate (tenured) professor @ U. Sherbrooke, 2023 – Now

Interested in theoretical CS

Interested in theoretical CS, e.g. I took these optional courses:

- Computational complexity theory
- Numerical algorithms
- Graph theory (@ math departement)
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- Flow networks
- Optimization
- Group representation theory (@ math departement)
- Advanced cryptography (@ McGill University)

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*Regret: not doing
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Possible to learn
more along the way!

B.Sc. and M.Sc.: grades

Grades (B.Sc.)

B.Sc. and M.Sc.: grades

Grades (B.Sc.)

A
C
D+
B+
A-

B.Sc. and M.Sc.: grades

Grades (B.Sc.)

A
C
D+
B+
A-

Too low for undergrad
research internship 😬

B.Sc. and M.Sc.: grades

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A
C
D+
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Grades (B.Sc.)

A
C
D+
B+
A-

A
A+
A+
B

3, 3

B.Sc. and M.Sc.: grades

Grades (B.Sc.)

A
C
D+
B+
A-

A
A+
A+
B



3, 3

B.Sc. and M.Sc.: grades

Grades (B.Sc.)

A
C
D+
B+
A-

A
A+
A+
B

3, 3

A+
A+
A+
A

4, 2

B.Sc. and M.Sc.: grades

Grades (B.Sc.)

A
C
D+
B+
A-

A
A+
A+
B

3, 3

A+
A+
A+
A



4, 2

B.Sc. and M.Sc.: grades

Grades (B.Sc.)

A
C
D+
B+
A-

A
A+
A+
B

3, 3

A+
A+
A+
A

4, 2

A+
A
A-
A+

A
A+
A+

A

4, 1

A
A+
A
A-

4, 0

B.Sc. and M.Sc.: grades

Grades (B.Sc.)

A
C
D+
B+
A-

A
A+
A+
B

3, 3

A+
A+
A+
A

4, 2

A+
A
A-
A+

A
A+
A+

A

4, 1

A
A+
A
A-

4, 0

Just enough for
scholarships 😊

B.Sc. and M.Sc.: grades

Grades (B.Sc.)

A
C
D+
B+
A-

A
A+
A+
B

3, 3

A+
A+
A+
A

4, 2

A+
A
A-
A+

A
A+
A+

A

4, 1

A
A+
A
A-

4, 0

Regret: not taking 1st
semester more seriously
(2.86/4.3 vs. 4.07/4.3 \rightarrow 3.82/4.3)

B.Sc. and M.Sc.: grades

Grades (B.Sc.)

A
C
D+
B+
A-

A
A+
A+
B

3, 3

A+
A+
A+
A

4, 2

A+
A
A-
A+

A
A+
A+

A

4, 1

A
A+
A
A-

4, 0

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Possible to make
your own way!

Initiation to research

- Undergrad research scholarship (B.Sc. last summer):
arithmetic circuits and proof trees



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*Outcome: read a dozen of papers/books on circuit complexity,
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- Research project (B.Sc. last semester):
complexity of the automata intersection problem



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Outcome: springboard for a master's



Master's thesis: complexity of the automata intersection problem

Master's thesis: complexity of the automata intersection problem



Master's thesis: complexity of the automata intersection problem



Results:

	Max size b of S_i ; max # of final states		
	1	2	3 or more
Single generator; $ \Sigma = 1$	L	L	NP
Elementary 2-groups	$\oplus L$	$\oplus L$	NP (Beaudry 1988b)
Elementary p -groups	$\text{Mod}_p L$	NP	NP (Beaudry 1988b)
All abelian groups	$\in \text{NC}^3, \in \text{FL}^{\text{ModL}}/\text{poly}$	NP	NP (Beaudry 1988b)

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	1	2	3 or more
Idempotent commutative	$\in \text{AC}^0$	NP	NP (Beaudry 1988b)
Groups	$\in \text{NC}$ (Luks 1990)	NP	NP
Commutative	NP (Beaudry <i>et al.</i> 1992)	NP (Beaudry <i>et al.</i> 1992)	NP (Beaudry <i>et al.</i> 1992)
Idempotent	NP (Beaudry <i>et al.</i> 1992)	NP (Beaudry <i>et al.</i> 1992)	NP (Beaudry <i>et al.</i> 1992)
Aperiodic	NP (Beaudry <i>et al.</i> 1992)	NP	NP (Beaudry <i>et al.</i> 1992)
All monoids	PSPACE (Kozen 1977)	PSPACE (Kozen 1977)	PSPACE (Kozen 1977)

Master's thesis: complexity of the automata intersection problem



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Outcome: CSR'12 + Computational Complexity'16

Applied for a scholarship to do this:

Super-quadratic lower bounds on the size of branching programs

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Instead, did this for > 6 months:



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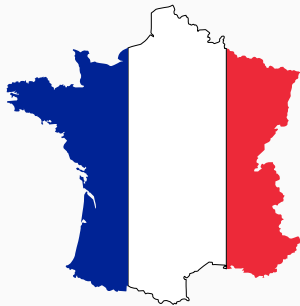


What to do with experience in theoretical CS and strikes?

Applied for a scholarship to do this:

Super-quadratic lower bounds on the size of branching programs

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What to do with experience in theoretical CS and strikes?

Ph.D.: naive plan

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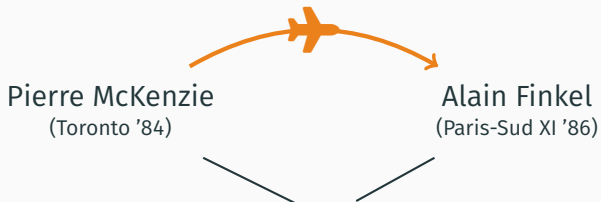
Super-quadratic lower bounds on the size of branching programs

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What to do with experience in theoretical CS and strikes?





Ph.D.: new horizons

Complexity

Stephen Cook
(Harvard '66)

Allan Borodin
(Cornell '69)

Pierre McKenzie
(Toronto '84)

Concurrency and verification

Guy Vidal-Naquet
(Paris-VI '81)

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Ph.D.: new horizons

Algorithms and complexity of *counter-based systems*

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Watch out:
finding an apartment
is the first open problem!



Ph.D.: contribution (~3 years)

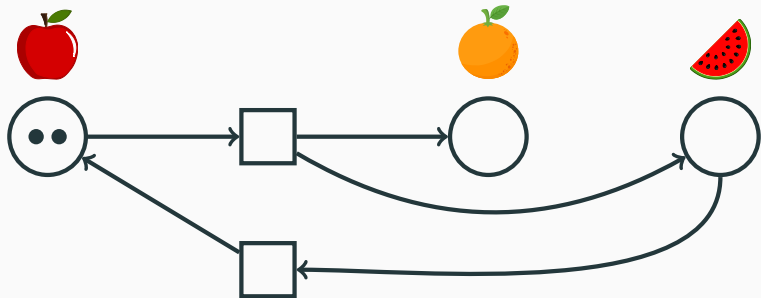
Conference papers

- M. Blondin, A. Finkel, P. McKenzie. *Handling Infinitely Branching WSTS*. ICALP 2014.
- M. Blondin, A. Finkel, S. Göller, C. Haase, P. McKenzie. *Reachability in 2D Vector Addition Systems with States is PSPACE-complete*. LICS 2015.
- M. Blondin, A. Finkel, C. Haase, S. Haddad. *Approaching the Coverability Problem Continuously*. TACAS 2016.

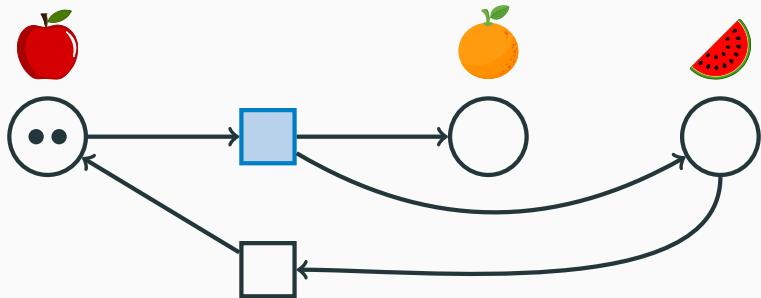
Journal papers

- M. Blondin, A. Finkel, C. Haase, S. Haddad. *The Logical View on Continuous Petri Nets*. ACM Transactions on Computational Logic (TOCL), 2017.
- M. Blondin, A. Finkel, P. McKenzie. *Well Behaved Transition Systems*. Logical Methods in Computer Science (LMCS), 2017.
- M. Blondin, A. Finkel, P. McKenzie. *Handling Infinitely Branching Well-structured Transition Systems*. Information and Computation, 2018.
- M. Blondin, M. Englert, A. Finkel, S. Göller, C. Haase, R. Lazić, P. McKenzie, P. Totzke. *The Reachability Problem for Two-Dimensional Vector Addition Systems with States*. Journal of the ACM (JACM), 2021.

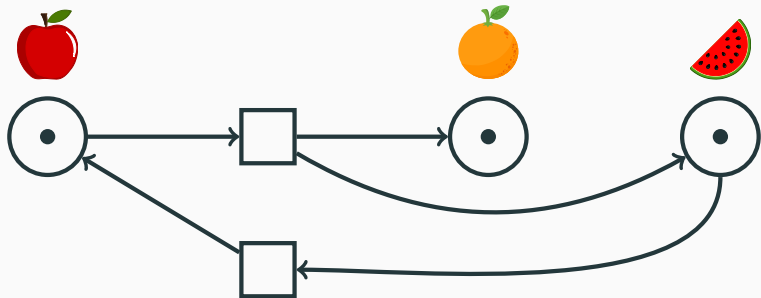
Ph.D.: reachability in Petri nets



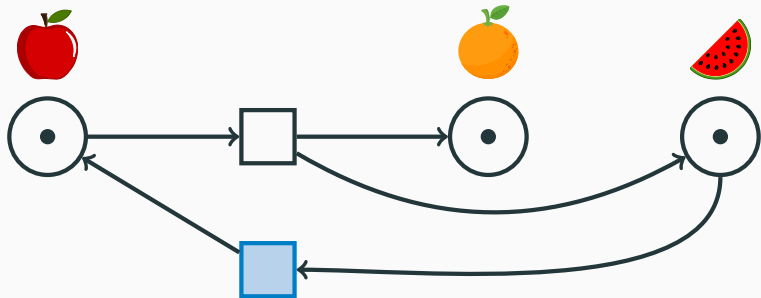
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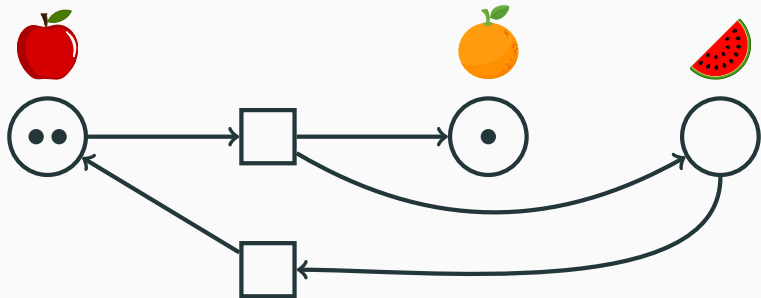
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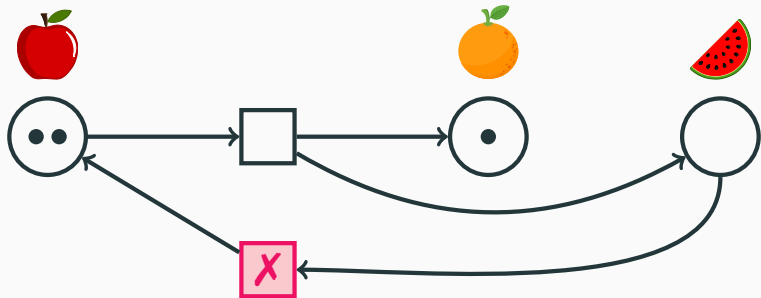
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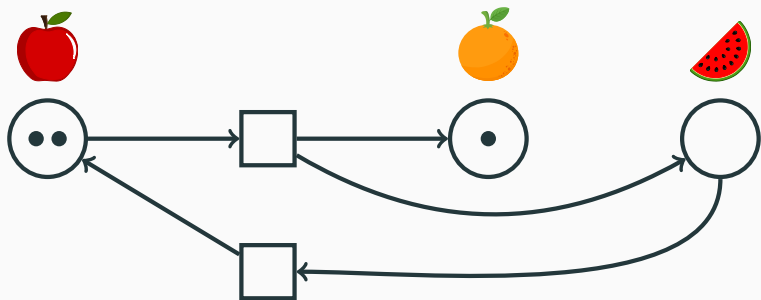
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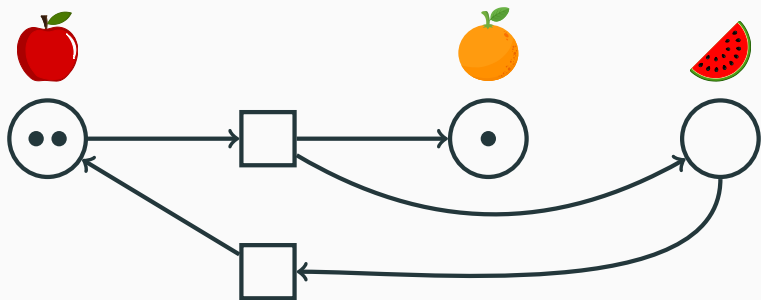
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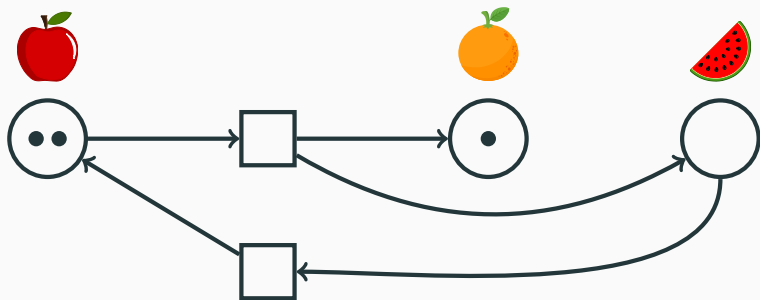
Ph.D.: reachability in Petri nets



$$\{\text{apple} : 2\} \xrightarrow{*} \{\text{apple} : 2, \text{orange} : 1\}$$



Reachability: $x \xrightarrow{*} y?$

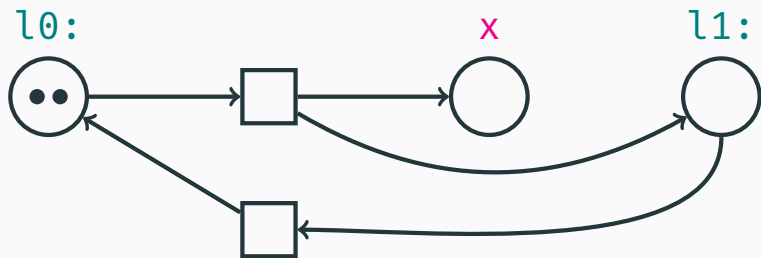


Useful for the formal verification of

- Concurrent programs
- Protocols
- Business processes
- Biological processes

⋮

Ph.D.: reachability in Petri nets



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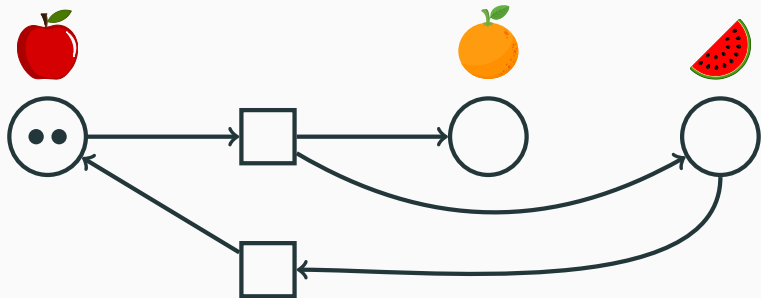
⋮

```
spawn proc()  
spawn proc()
```

```
proc():
```

```
l0:    x++  
l1:    goto l0
```

Ph.D.: reachability in Petri nets



Useful for the formal verification of

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*I did not care
much about this...*

Ph.D.: reachability in Petri nets



Quanta magazine

Physics

Mathematics

Biology

Computer Science

Topics

Archive



COMPUTATIONAL COMPLEXITY

An Easy-Sounding Problem Yields Numbers Too Big for Our Universe



Researchers prove that navigating certain systems of vectors is among the most complex computational problems.



Ph.D.: reachability in Petri nets

In general

EXPSPACE-hard (1976)

Decidable (1981–2011)

Two unbounded counters

Decidable (1979)

$\in 2\text{-EXPTIME}$ (1986)

Ph.D.: reachability in Petri nets

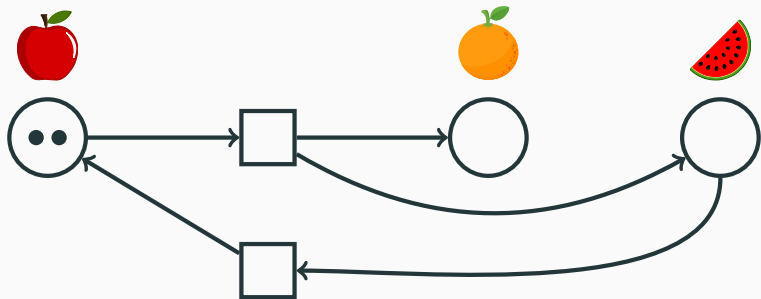
In general	Two unbounded counters
EXPSPACE-hard (1976)	
	Decidable (1979)
Decidable (1981–2011)	
	$\in 2\text{-EXPTIME}$ (1986)
$\in \text{Ackermann}$ (2015)	PSPACE-complete (2015)

Ph.D.: reachability in Petri nets

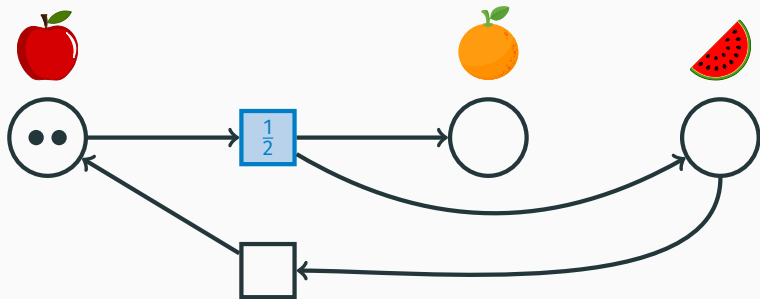
In general	Two unbounded counters
EXPSPACE-hard (1976)	
	Decidable (1979)
Decidable (1981–2011)	
	$\in 2\text{-EXPTIME}$ (1986)
$\in \text{Ackermann (2015)}$	PSPACE-complete (2015)
$\mathcal{O}\left(2^{\cdot^{\cdot^{\cdot^2}}}\right)$ -hard (2019)	
Ackermann-hard (2021)	

$A(5) > \# \text{ atoms in the universe}$

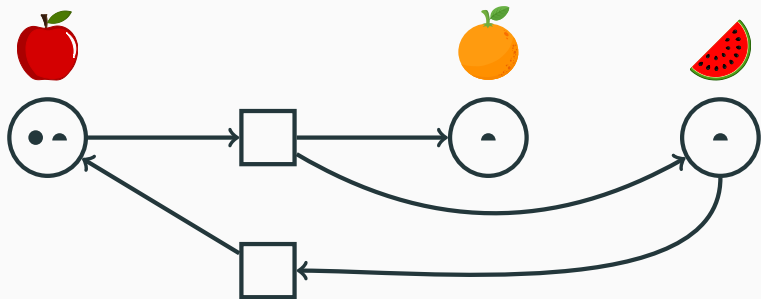
Ph.D.: continuous reachability Petri nets

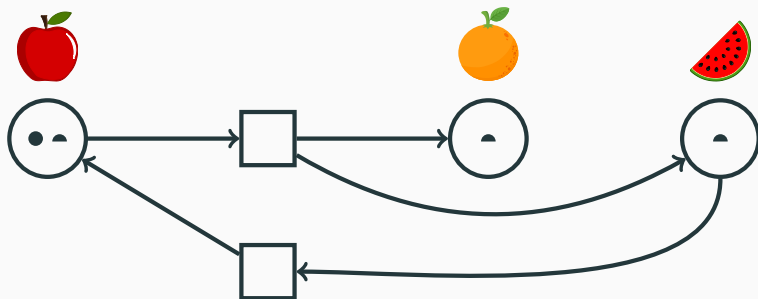


Ph.D.: continuous reachability Petri nets



Ph.D.: continuous reachability Petri nets





Continuous reachability is solvable in polynomial time

Approaching the Coverability Problem Continuously

Michael Blondin^{1,2*}, Alain Finkel², Christoph Haase^{2**}, and Serge Haddad^{2,3***}

¹ DIRO, Université de Montréal, Canada

² LSV, CNRS & ENS Cachan, Université Paris-Saclay, France

³ Inria, France

Abstract. The coverability problem for Petri nets plays a central role in the verification of concurrent shared-memory programs. However, its high EXPSpace-complete complexity poses a challenge when encountered in real-world instances. In this paper, we develop a new approach to this problem which is primarily based on applying forward coverability in continuous Petri nets as a pruning criterion inside a backward-coverability framework. A cornerstone of our approach is the efficient encoding of a recently developed polynomial-time algorithm for reachability in continuous Petri nets into SMT. We demonstrate the effectiveness of our approach on standard benchmarks from the literature, which shows that our approach decides significantly more instances than any existing tool and is in addition often much faster, in particular on large instances.

Approaching the Coverability Problem Continuously

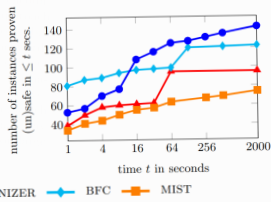
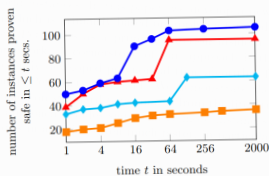
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Abstract. The coverability problem for Petri nets plays a central role in the verification of concurrent shared-memory programs. However, its high EXPSPACE-complete complexity poses a challenge for its application in real-world scenarios. In this paper, we propose a novel approach to this problem by encoding it in the context of continuous reachability. This encoding shows that the coverability problem can be reduced to a continuous reachability problem, which can be solved using existing tool instances.



Ph.D.: continuous reachability Petri nets

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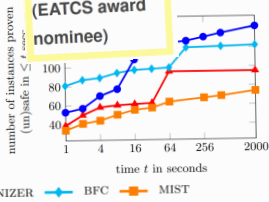
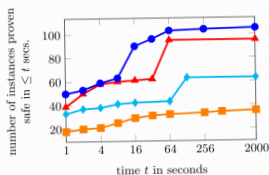
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*Approaching the
Coverability
Problem
Continuously
(EATCS award
nominee)*

Abstract. The coverability problem for Petri nets plays a central role in the verification of concurrent shared-memory programs. However, it is PSPACE-complete. We show that the problem is tractable in the continuous semantics. We present a new encoding of the problem in continuous Petri nets. We show that this encoding is more efficient than existing tool instances.



Ph.D.: continuous reachability Petri nets

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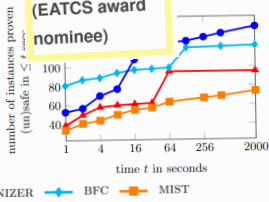
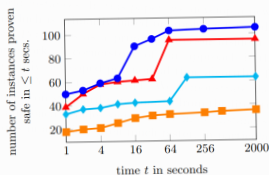
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Okay, maybe
I should care
about applications!

Abstract. The coverability problem for Petri nets plays a central role in the verification of concurrent shared-memory programs. However, it is EXPTIME-complete complexity. We propose a new approach to this problem in the context of continuous reachability encoding of Petri nets. In this paper, we show that our approach is more efficient than existing tool instances.



End of Ph.D.: now what?

- Plan: apply 1 year ahead for a scholarship
to go to Oxford (Oct. 2, 2015)

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- Ph.D. thesis submission (Apr. 22, 2016)

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*PhD and tenure are the hardest
Finding a postdoc is the easiest
— Colleague*

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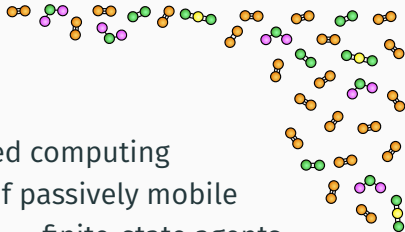
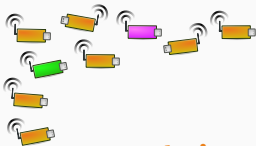
First open problem:
Obtaining visas

Second open problem:
Finding an apartment



Population protocols: distributed computing
model for massive networks of passively mobile
finite-state agents

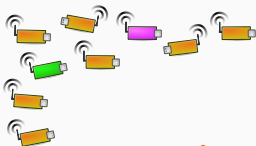
Postdoc: population protocols



Population protocols: distributed computing
model for massive networks of passively mobile
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Model e.g. networks of passively **mobile sensors** and
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Postdoc: population protocols



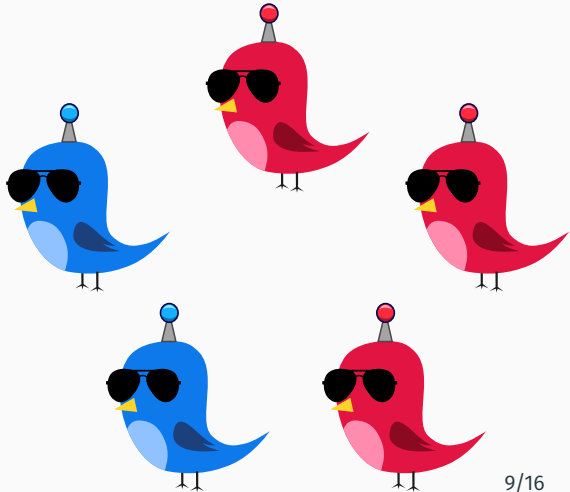
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Model e.g. networks of passively **mobile sensors** and
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Protocols **compute predicates** of the form $f: \mathbb{N}^d \rightarrow \{0, 1\}$
e.g. $f(m, n)$ is computed by $m + n$ agents

Postdoc: population protocols (majority)

red agents \geq # blue agents?

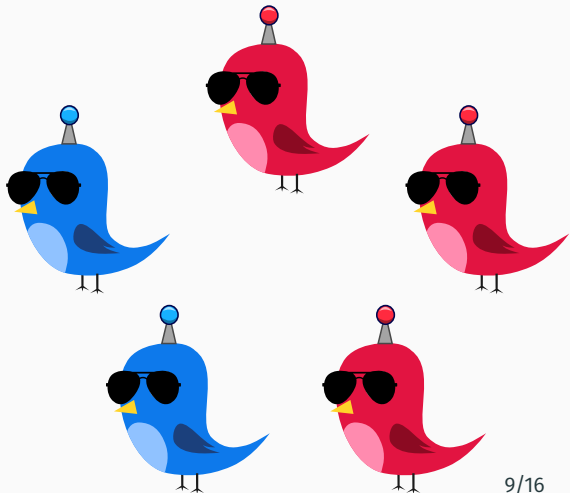


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- Large agents convert small agents to their colour

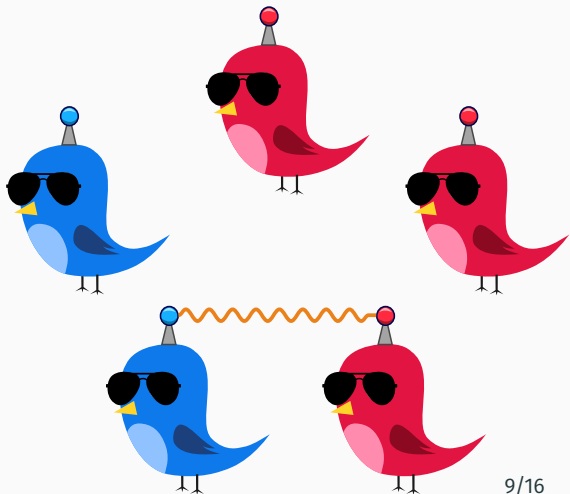


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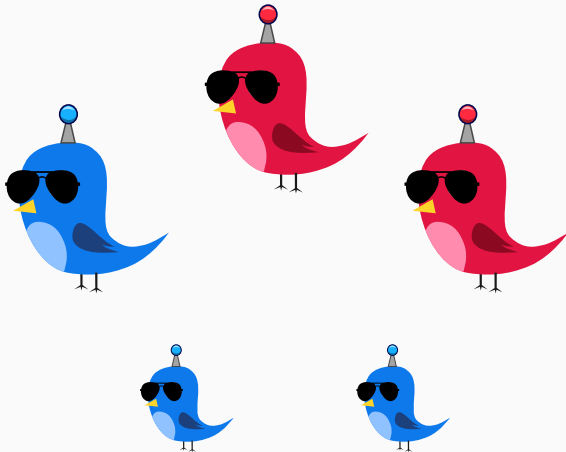


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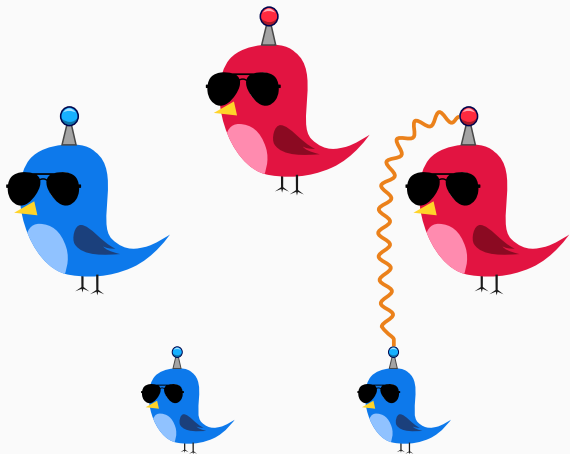


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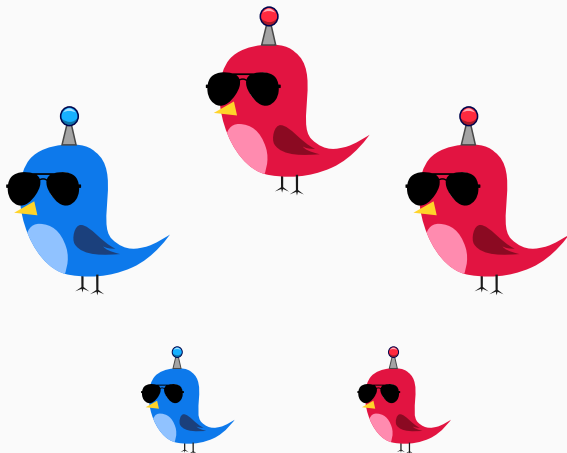


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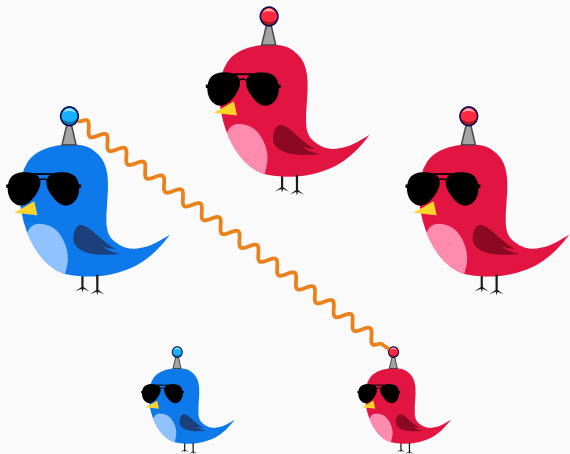


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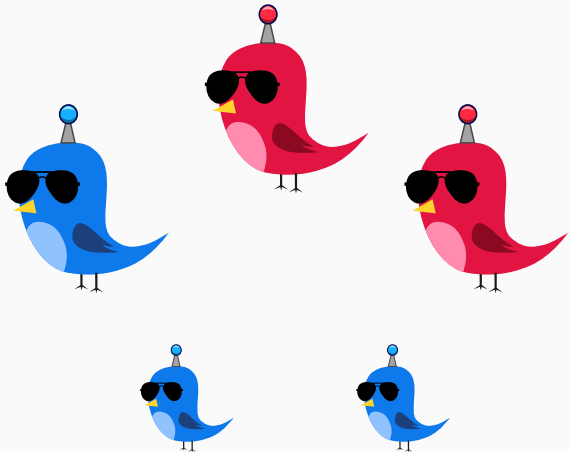


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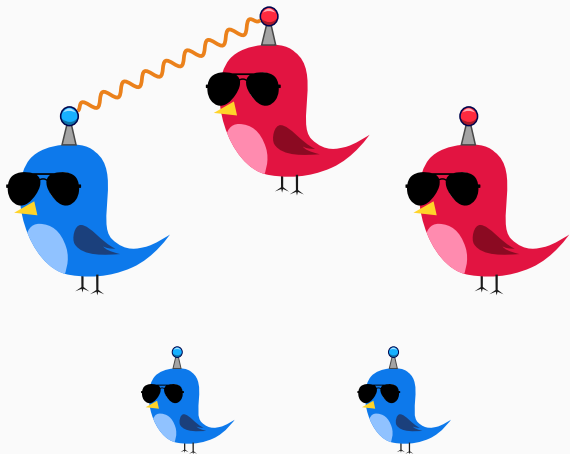


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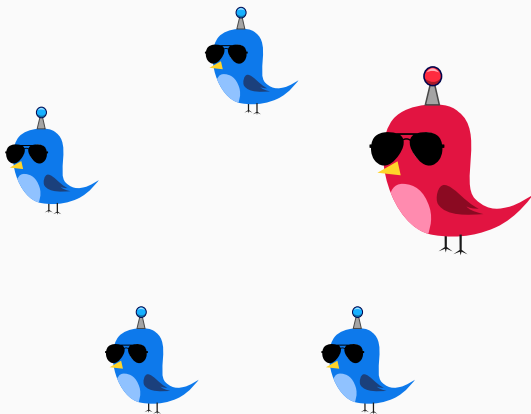


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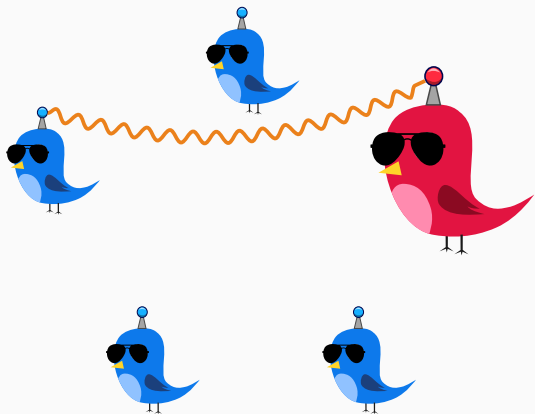


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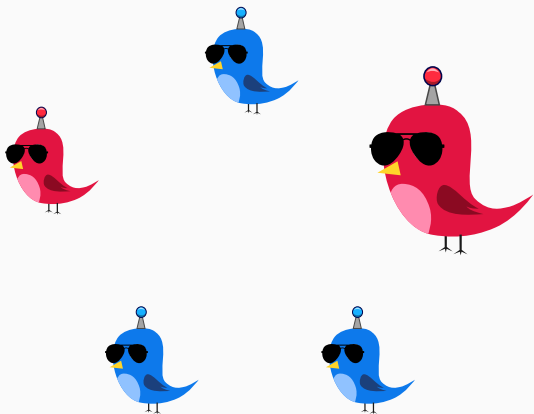


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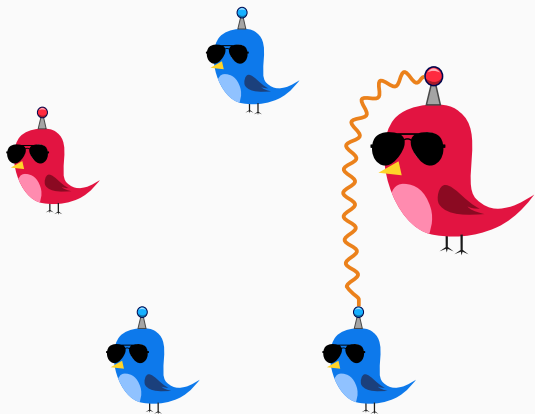


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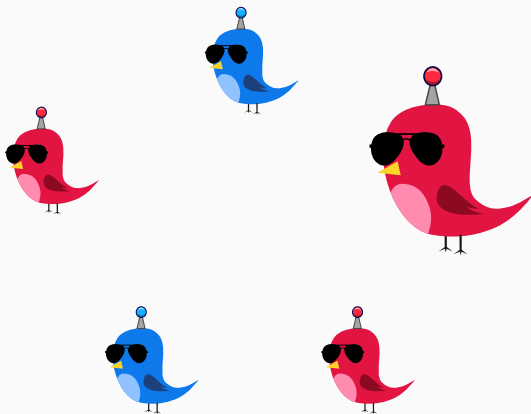


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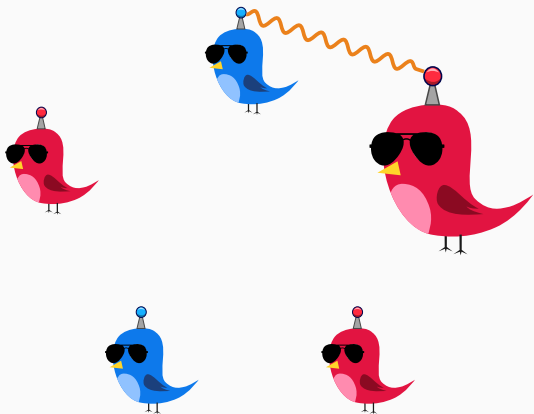


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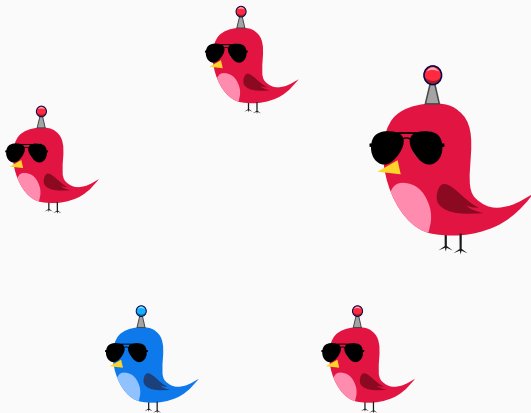


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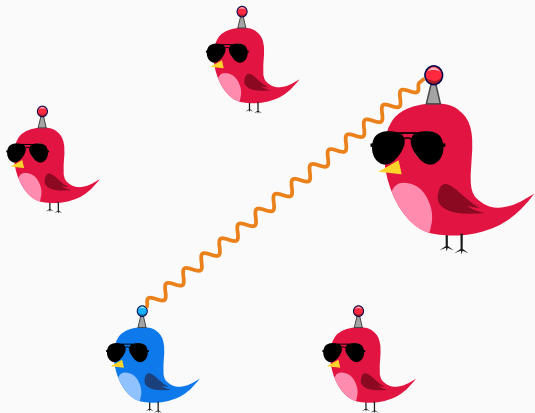


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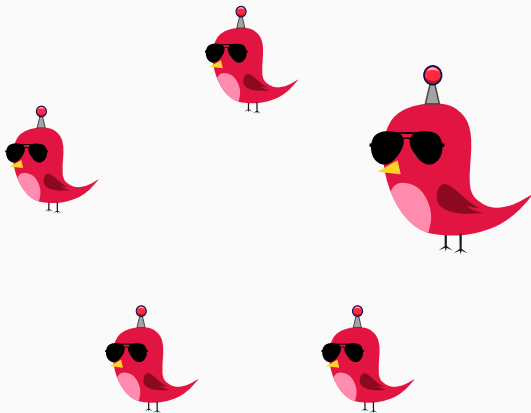


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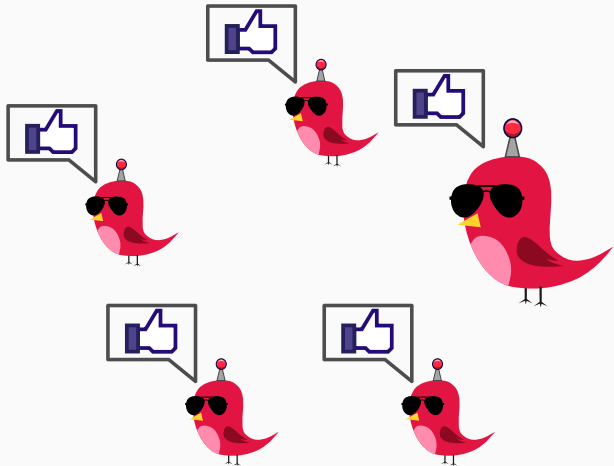


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- Formally and automatically verify that a protocol works as intended (PODC'17)
- Automatic analysis of the expected running time of a protocol (CONCUR'18)
- Tool support (CAV'18)
- State complexity of protocols (STACS'18)


Peregrine

A tool for the analysis of population protocols

 Analyze an existing protocol

 Create a new protocol

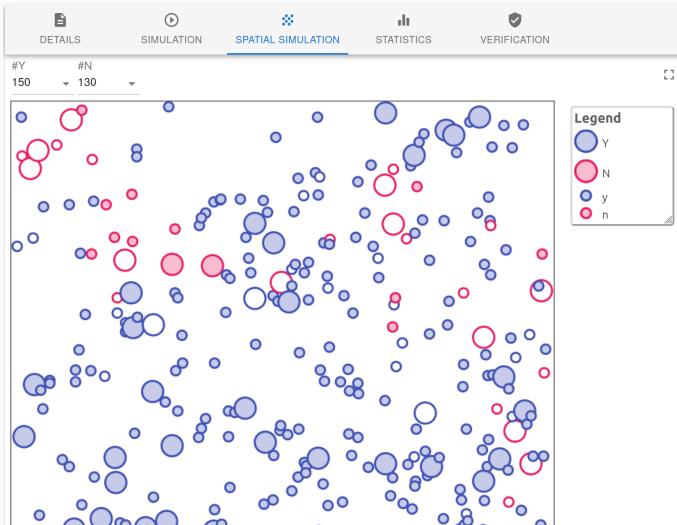
 Import an existing protocol

New to Peregrine? **Read the documentation** 

Postdoc: work

Majority Voting protocol

Description: This protocol takes a majority vote. More precisely, it computes whether there are initially more agents in state Y than N.



Postdoc: work

DETAILS

SIMULATION

SPATIAL SIMULATION

STATISTICS

VERIFICATION

Property to be verified: Correctness (with stage graphs) ▾ ✓ VERIFY

✓

The protocol satisfies correctness.

The expected number of interactions until a stable consensus is reached is $2^{O(n \log n)}$.

Pick custom initial configuration:

#Y

#N

1 ▾

1 ▾

↻ RESET

Current configuration (●):

N

Y

⏮ PREV

▶ PLAY

⏭ NEXT

▲ PROGRESS (1)

⌵

☰

⦿

S0

S4

S5

S1

S2

S3

●

Stage S1

Speed: $\mathcal{O}(n^2 \log n)$

Certificate: $C[N]$

Certificate Value: 1

Constraint: $\text{PotReach}(C[Y] \geq C[N])$

Eventually dead transitions (2):

Y

N

↔

y

n

N

y

↔

N

n

Postdoc: contribution (~2 years)

- Work with former Ph.D. advisor on WSTS: FSTTCS'17
- Work with collaborators on Petri net relaxations:
LICS'17, CONCUR'18
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Nonregret: going to Munich!

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*Tip: do something at least a bit
different during a postdoc*

Prof: finding a position

- Wanted to come back to Quebec
(family, friends, partner, language, sense of duty, etc.)
- Restricted pool of 3-5 interesting universities
- Luck: U. Sherbrooke opened a position one year into my postdoc

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Only 2 positions in TCS
across Quebec in 8 years

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*Watch out for such
narrow expectations!*

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*Beware of
scientific isolation!*

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- Twice TA + supervised undergrad project in Munich (2017)

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His publications during Ph.D.:
4 x LICS, 2 x CAV, 1 x TACAS + 2 x journals

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Tried to pull this a second time, but student went to ENS Paris-Saclay (and then Oxford)

- *Formal verification methods for the development of reliable dynamic networks*
- *Efficient verification of concurrent and distributed infinite-state systems*
- *Automatic incorrectness analysis of counter-based systems*

Prof: scientific contribution (~6 years)

- State complexity, expressiveness and verification of **population protocols** (CONCUR'19, STACS'20, CAV'20)

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(LICS'20, TACAS'21, LICS'21, FoSSaCS'22, LICS'23
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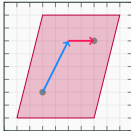
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For example, $M = \{m_1, m_2, m_3\}$ where



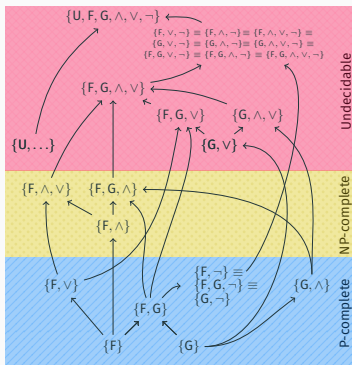
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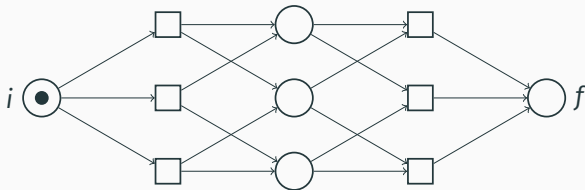
Safe scheduling	Safe reachability	Safe planning	etc.
GZ	Z U target	$(\neg Z_1 \wedge \dots \wedge \neg Z_4)$ U target	
			

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- Population protocols with **unordered data** (ICALP'23)



e.g. some color has absolute majority?

Prof: scientific contribution (~6 years)

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- **Data structures** for formal verification (TACAS'25)

Prof: scientific contribution (~6 years)

- State complexity, expressiveness and verification of **population protocols** (CONCUR'19, STACS'20, CAV'20)

- **Relaxations of Petri nets**

(L

+

Students on the papers mentioned:

- M.Sc. → data engineer
- Ph.D. → R&D @ Informal Systems (4)
- M.Sc. → programmer
- P (23)

- **Data structures** for formal verification (TACAS'25)

Prof: my tenure application after 4 years

- **Publications:** $9 \times \text{conf.} + 6 \times \text{journals} + 1 \times \text{invited} + 1 \times \text{book}$
- **Mentoring:** $1 \times \text{Ph.D.} + 3 \times \text{M.Sc.} + 4 \times \text{interns} + 10 \times \text{projects}$
- **Talks:** 9

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- **Reviewing:** $3 \times \text{program committees} + 18 \text{ reviews}$
- **Administrative:** $7 \times \text{committees} + \text{president} + \text{union rep.}$

I think this is a sign that I don't take enough risks:

	Conf.	Journals
# submissions	27	12
# accepted	24	12
# rejected	3	0

I think this is a sign that I don't take enough risks:

	Conf.	CORE ranking
# submissions	27	
# accepted	24	11 \times A*, 11 \times A, 2 \times national
# rejected	3	3 \times A*

I think this is a sign that I don't take enough risks:

	Conf.	CORE ranking
# submissions	27	
# accepted	24	$11 \times A^*$, $11 \times A$, $2 \times \text{national}$
# rejected	3	$3 \times A^*$

Tip: take some risks!

I think this is a sign that I don't take enough risks:

	Conf.	CORE ranking
# submissions	27	
# accepted	24	11 \times A*, 11 \times A, 2 \times national
# rejected	3	3 \times A*

But: science takes time!

Teaching: back as a student

Teaching assistant:

Degree	# times as TA	# distinct courses
B.Sc.	2	2
M.Sc.	5	3
Ph.D.	8	4
Postdoc	6	4
Total	21	11

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Teaching assistant:

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B.Sc.	2	2
M.Sc.	5	3
Ph.D.	8	4
Postdoc	6	4
Total	21	11

Time consuming!
*+ I really liked to write
detailed exercise sheets*

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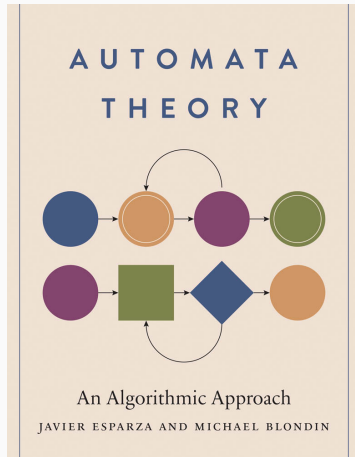
Prize for best TA of U. Montreal

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Teaching assistant:

Degree	# times as TA	# distinct courses
B.Sc.	2	2
M.Sc.	5	3
Ph.D.	8	4
Postdoc	6	4
Total	21	11

*Not needed to get a position!
(but a bit of experience won't hurt)*



MIT Press, 2023 

Teaching: now

Programmation système

Notes de cours (IFT200)

```
• 01001001
  01000110
  01010100
  00110010
  00010000
  00111001
  00000000
```

Michael Mendler



Algorithmes et structures de données

Notes de cours (IFT430)

Michael Mendler



Techniques de vérification et de validation

Notes de cours (ICS502/752)

Michael Mendler



Théorie du calcul

Notes de cours (IFS03/711)

Michael Mendler

Teaching: now



≥ 764 pages, 336 exercises, 275 sol.
100 code implem. with visual demo.
77 videos, 14 quizzes, etc.

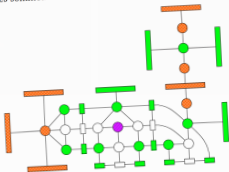
Teaching: now



La carte contient

- une route complète de taille 2 entre une cité et un village (A2 – B2);
- une route incomplète de taille 1 à partir d'un village (B2);
- une route incomplète de taille 2 à partir d'un village (B2 – C2);
- une cité incomplète de taille 1 (A2);
- une cité complète de taille 2 (C1 – C2);
- une cité incomplète de taille 1 (C1).

La carte se modélise à l'aide d'un graphe non dirigé où l'on représente les frontières d'une tuile par des sommets rectangulaires, les attributs d'une tuile par des sommets circulaires, et leurs liens par des arêtes:



**Techniques de vérification
et de validation**

Notes de cours (ICS502/752)

Michael Mendler



Théorie du calcul

Notes de cours (IFS90/711)

Michael Mendler

La carte contient

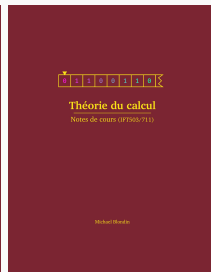
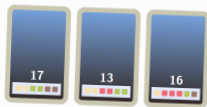
- une route
- une route
- une route
- une cité
- une cité
- une cité

La carte se situe
à la frontière de
deux régions de



La première carte du bas procure deux unités de curcuma. La deuxième carte du bas permet d'échanger trois unités de curcuma contre une unité de cardamome et une unité de safran. La carte au centre tout à droite permet d'améliorer (au plus) deux unités (de n'importe quel type), par ex. transformer deux unités de curcuma vers deux unités de safran, ou bien une unité de curcuma vers une unité de cardamome.

Les épices permettent d'acheter des cartes points comme celles-ci:



Teaching: now

CHAPITRE 1 SOLUTIONS DES EXERCICES

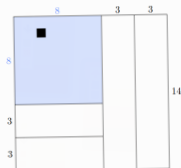
180

La carte contient

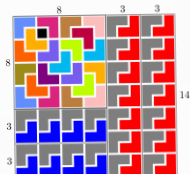
- une route
- une route
- une route
- une cité
- une cité
- une cité

La carte se n
frontières d'
tuile par de

La prem
carte du
de card;
permet
ex. trans
bien une
Les épice



Nous avons vu à la section 0.2.4 que la sous-grille 8×8 est quasi-pavable puisque 8 est une puissance de 2. De plus, par (a), il est possible de paver une sous-grille de taille $3 \times k$ où k pair. Par exemple, sur l'exemple ci-dessus, nous obtenons ce pavage:



0 1 2 3 4 5 6 7 8 9

Théorie du calcul

Notes de cours (IFS90/711)

Michael Mendicino

Teaching: now

CHAPITRE 1 SOLUTIONS DES SOLUTIONS DES EXERCICES

169

La carte contient

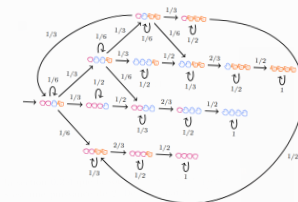
- une route
- une route
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- une cité
- une cité

La carte se n
frontières d'
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La prem
carte du
de card;
permet
ex. trans
bien une

Les épice

Nou
pavi
pos
sur



Nous avons;

$$\begin{aligned}
 \mathbb{P}(\text{sequence of nodes}) &= \sum_{i=1}^{\infty} \left(\left(\sum_{j=0}^{\infty} (1/6)^j \cdot (1/3) \right)^3 \right)^i \\
 &= \sum_{i=1}^{\infty} \left((6/5 \cdot 1/3)^3 \right)^i \\
 &= \sum_{i=1}^{\infty} (8/125)^i \\
 &= \sum_{i=0}^{\infty} (8/125)^i - 1 \\
 &= (125/117) - 1 \\
 &= 8/117.
 \end{aligned}$$

0 1 2 3 4 5 6 7 8 9

Théorie du calcul

Notes de cours (IFS90/711)

Michael Ward

Teaching: now

La carte contie

- une route
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- une cit 
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- une cit 

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Les  pice

Nou
pavi
pos
sur

Nous

p

Consid rons le programme ci-dessous. Celui-ci cherche   d terminer si un nombre x est premier en essayant tous les diviseurs possibles de 2 jusqu'  $\lfloor \sqrt{x} \rfloor$. Nous allons d montrer formellement que le programme est correct malgr  l'usage risqu  de nombres en virgule flottante.

```
bool est_premier(uint64_t x)
{
    if (x <= 1)
        return false;

    double y = x;
    double z = sqrt(y);
    uint64_t d = 2;

    while (d <= z) {
        if (x % d == 0)
            return false;

        d++;
    }

    return true;
}
```

Dans le reste du document, nous faisons ces hypoth ses standards :

- « sqrt » est conforme   la norme IEEE 754;
- les valeurs de type **double** sont repr sent es en nombre en virgule flottante double pr cision binaire de la norme IEEE 754, avec le mode d'arrondi qui approxime au nombre le plus pr s, o  les bris d' galit  se font vers le nombre dont le dernier bit est pair.

$$\begin{aligned} &= (125/117) - 1 \\ &= 8/117. \end{aligned}$$

Teaching: now

CHAPITRE 1 SOLUTIONS DES SOLUTIONS DES EXERCICES

La carte contie

- une route
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Considérons !
nombre x est pre
Nous allons démo
risqué de nombr

```
bool est_pre
{
    if (x <= 1)
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    double
    double
    uint64_t

    while (d
        if (x
            retu

    d++;
}

return
}
```

Dans le
— « sq
— les
tant
ron
ver

Démonstration. Par la proposition 1, nous avons $|(x - \bar{x})/x| \leq 2^{-n/2}$. Puisque $x \geq 1$, cela implique que $|x - \bar{x}| \leq x \cdot 2^{-n/2}$. Comme $x < 2^{n+1}$, on en conclut que $|x - \bar{x}| < 2^{n/2}$. \square

Établissons maintenant une borne sur l'écart entre la racine carrée de x et la racine carrée de son approximation.

Proposition 3. Soient $n \in \mathbb{N}$ et $x \in \mathbb{R}^+ \cap [2^n, 2^{n+1})$. Nous avons $|\sqrt{x} - \sqrt{\bar{x}}| < 2^{(n/2)-52}$.

Démonstration. Nous avons

$$\begin{aligned} |\sqrt{x} - \sqrt{\bar{x}}| &= \frac{|x - \bar{x}|}{\sqrt{x} + \sqrt{\bar{x}}} && (\text{car } (a-b)(a+b) = a^2 - b^2) \\ &< \frac{2^{n-52}}{\sqrt{x} + \sqrt{\bar{x}}} && (\text{par le corollaire 2}) \\ &\leq \frac{2^{n-52}}{\sqrt{x} + \sqrt{x - 2^{n-52}}} && (\text{car } x \geq x - 2^{n-52} \text{ par le corollaire 2}) \\ &\leq \frac{2^{n-52}}{\sqrt{2^n} + \sqrt{2^n - 2^{n-52}}} && (\text{car } x \geq 2^n) \\ &\leq \frac{2^{n-52}}{\sqrt{2^n} + \sqrt{2^{n-1}}} && (\text{car } 2^n - 2^{n-52} \geq 2^{n-1}) \\ &\leq \frac{2^{n-52}}{2 \cdot \sqrt{2^{n-1}}} && (\text{car } 2^n \geq 2^{n-1}) \\ &= \frac{2^{n-52}}{2^{(n+1)/2}} \\ &\leq \frac{2^{n-52}}{2^{(n/2)+1}} \\ &= 2^{(n/2)-52}. \end{aligned}$$

Théorème 4. Soit $n \in \mathbb{N}$ tel que $1 \leq |n/2| \leq 51$ et soit $x \in \mathbb{R}^+ \cap [2^n, 2^{n+1})$. Nous avons $\sqrt{x} \geq \lfloor \sqrt{x} \rfloor$. \square

Démonstration. Par définition, \bar{x} se représente exactement en nombre en virgule flottante double précision. Ainsi, selon la norme IEEE 754, \sqrt{x} est égal au nombre que l'on obtient en calculant d'abord \sqrt{x} avec précision infinie, puis en l'arrondissant.

Remarquons que $\sqrt{x} \geq \sqrt{2^n} = 2^{n/2} \geq 2^{\lfloor n/2 \rfloor}$ et $\sqrt{x} < \sqrt{2^{n+1}} = 2^{(n+1)/2} \leq 2^{\lfloor n/2 \rfloor + 1}$. Ainsi, en représentation en virgule flottante infinie (binaire), \sqrt{x} peut s'écrire de cette forme :

$$1.d_1d_2 \dots \times 2^{\lfloor n/2 \rfloor}, \quad (1)$$

Curse or passion?

Teaching: now

CHAPITRE 1 SOLUTIONS DES SOLUTIONS DES EXERCICES

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$$1.d_1d_2 \dots \times 2^{(n/2)}, \quad (1)$$

Cons:

- Might hurt research
- or your personal life

Teaching: now

CHAPITRE 1 SOLUTIONS DES SOLUTIONS DES EXERCICES

La carte contie

- une route
- une route
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    double
    uint64_t

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        if (x
            retu

    }
    d++;
}
return
```

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— « sq
— les
tant
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ver

Pros:

- Academia also about education
- Grad student recruitment
- May spark research ideas
- **Fun!**

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Cons:

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- or your personal life

Teaching: now

CHAPITRE
SOLU

La carte contie
— une route
— une routi



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bord \sqrt{x} avec précision infinie, puis en
 $2^{(n/2)}$ et $\sqrt{x} < \sqrt{2^{n+1}} = 2^{(n+1)/2} \leq$
le flottante infinie (binaire), \sqrt{x} peut

Remember
to have fun! 😊

Thank you!