Formal Analysis of Population Protocols

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Antonín Kučera and Philipp J. Meyer

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Overview

Population protocols: distributed computing model for massive networks of passively mobile finite-state agents

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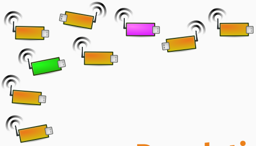

Population protocols: distributed computing model for massive networks of passively mobile finite-state agents

Can model *e.g.* networks of passively mobile sensors and chemical reaction networks

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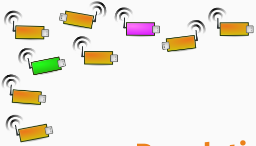

Population protocols: distributed computing model for massive networks of passively mobile finite-state agents

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Protocols compute predicates of the form $\varphi\colon \mathbb{N}^d \to \{0, 1\}$

e.g. if φ is unary, then $\varphi(n)$ is computed by n agents

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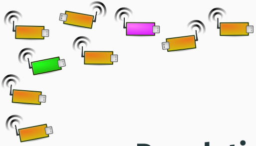

Population protocols: distributed computing model for massive networks of passively mobile finite-state agents

This talk: overview of recent advances on the formal analysis of population protocols

- anonymous mobile agents with very few resources
- agents change states via random pairwise interactions
- each agent has opinion true/false
- computes by stabilizing agents to some opinion

Population protocols

Angluin *et al.* PODC'04

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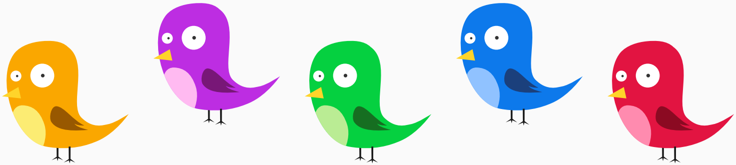

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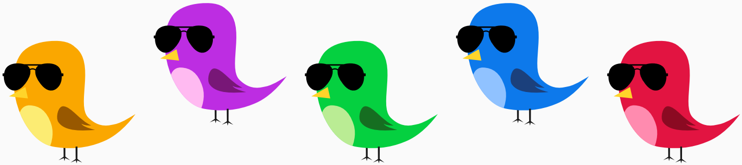

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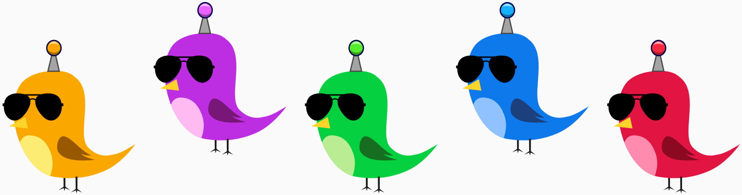

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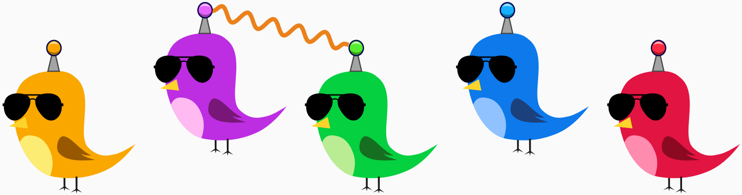

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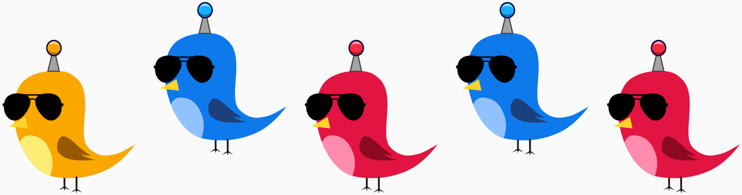

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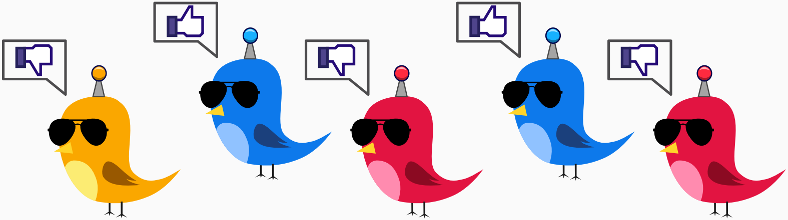

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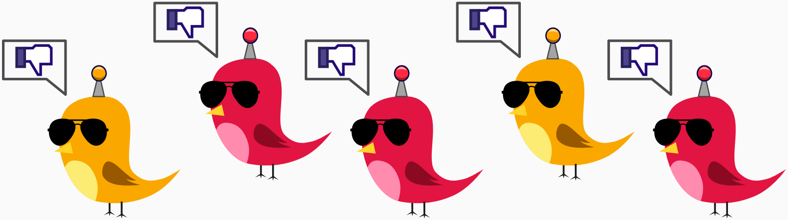

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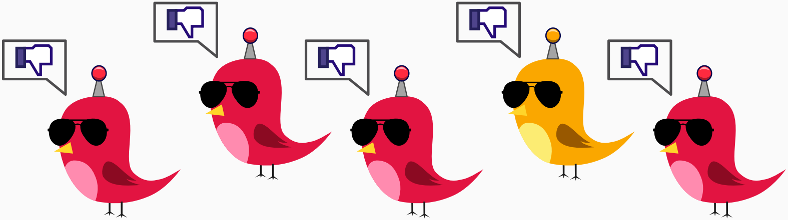

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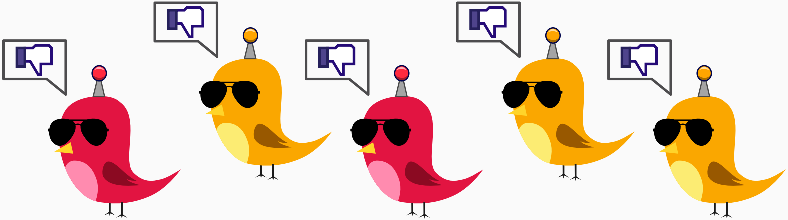
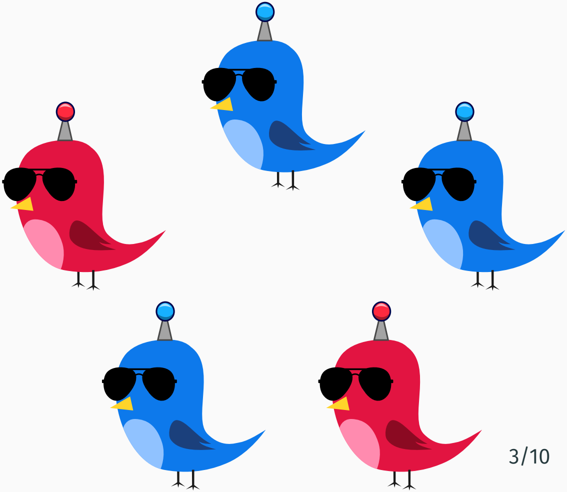


Example: majority protocol

At least as many blue birds than red birds?



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- Two large birds of different colors become small and blue
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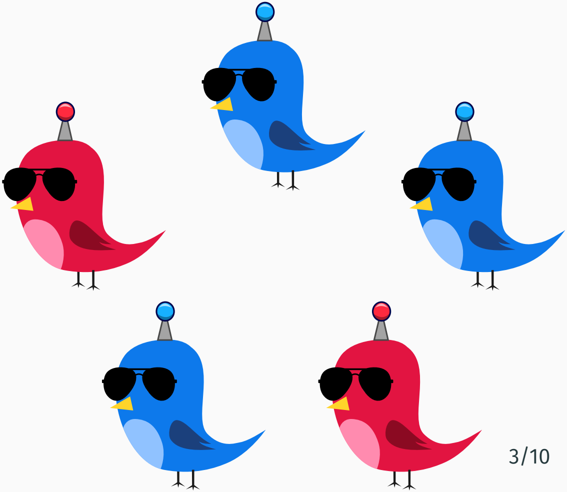

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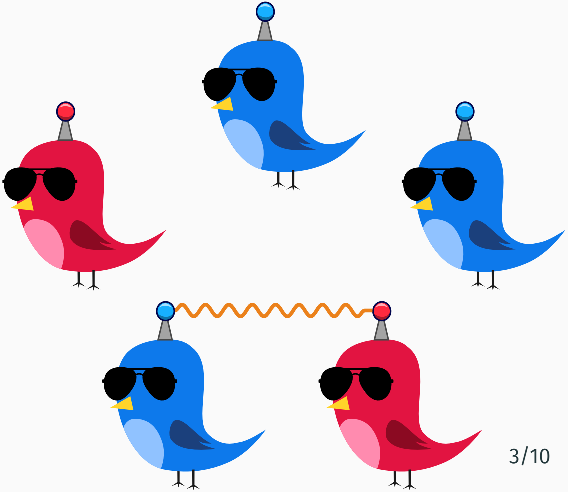

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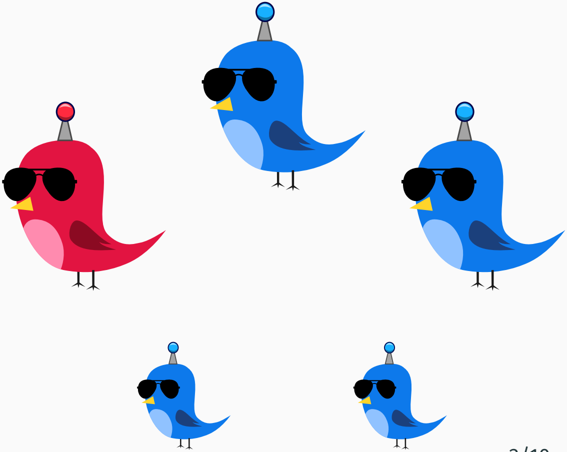

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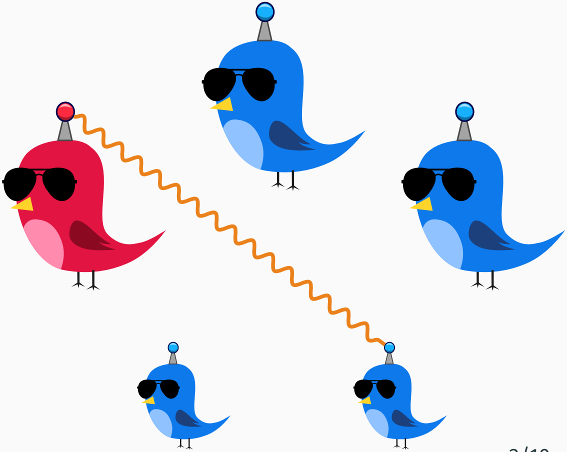

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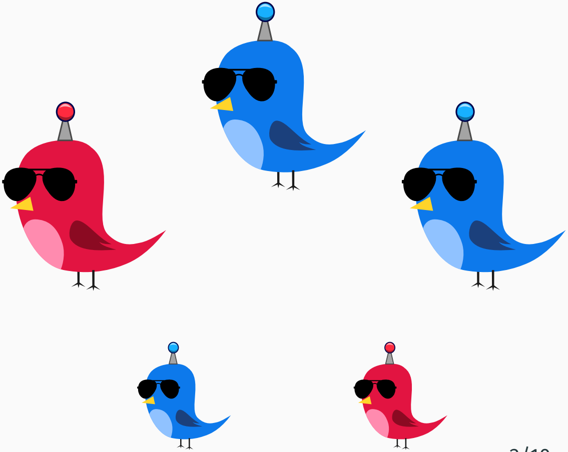

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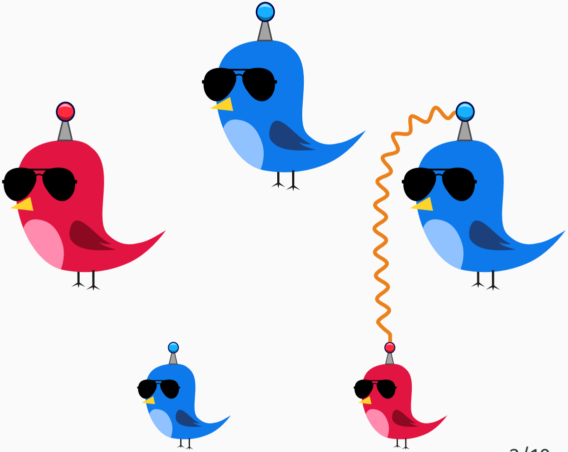

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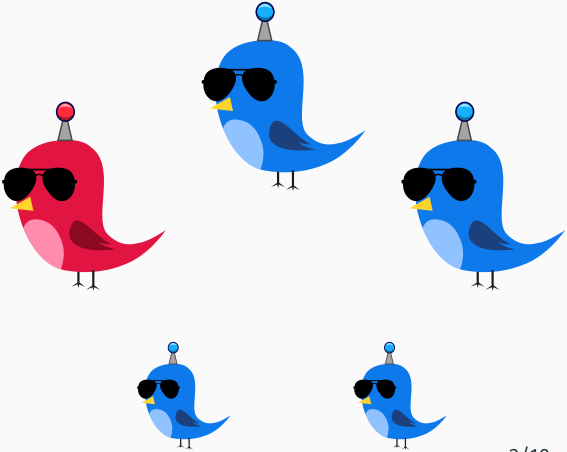

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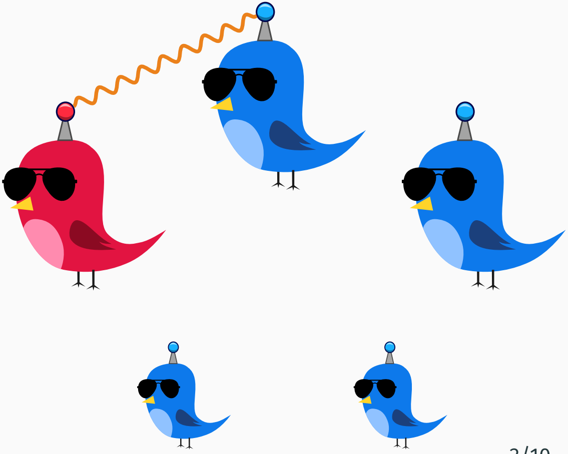

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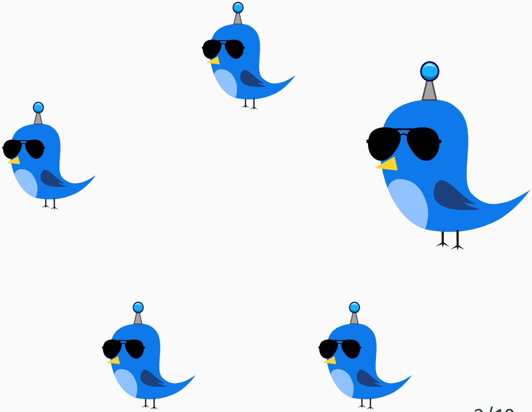

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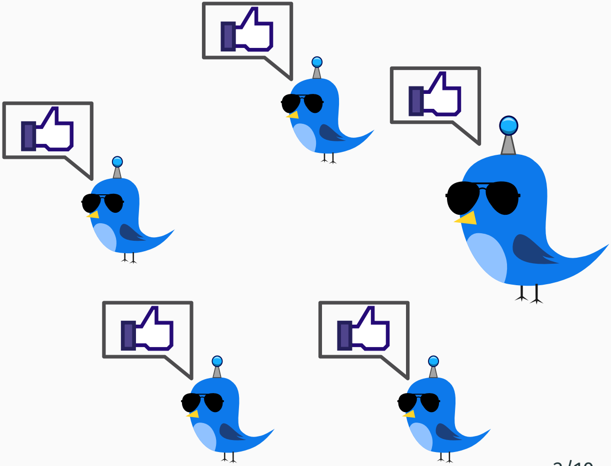

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At least as many blue birds than red birds?

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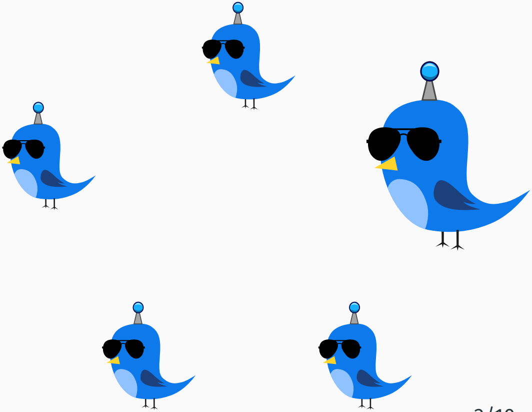
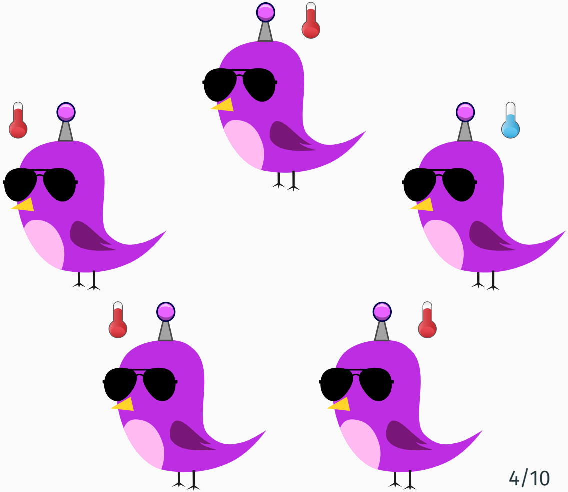
- **To break ties:** small blue birds convert small red birds



Example: threshold protocol

Are there at least 4 sick birds?



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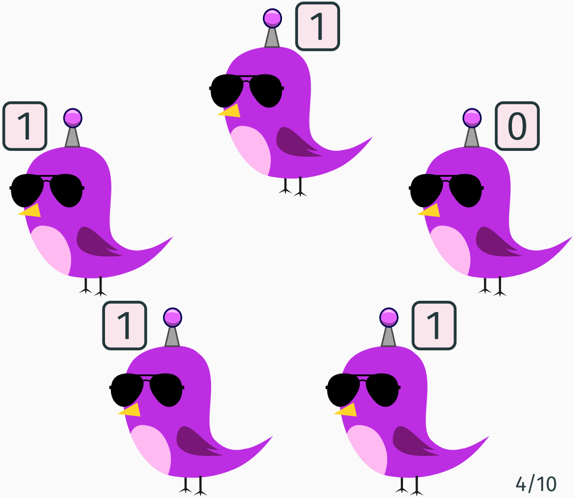

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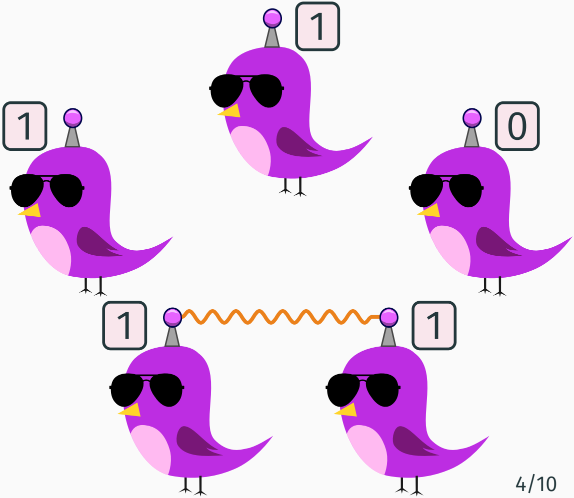

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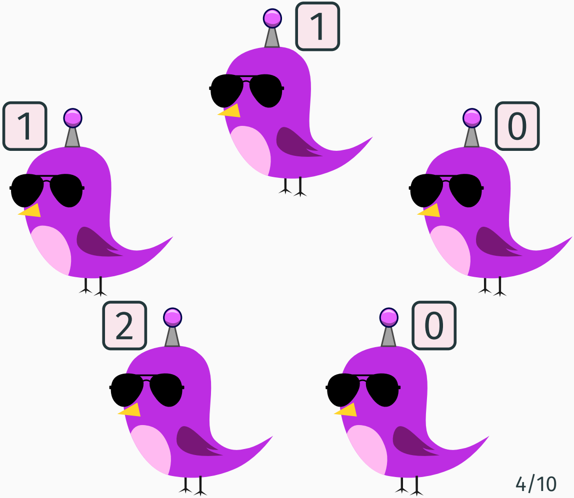

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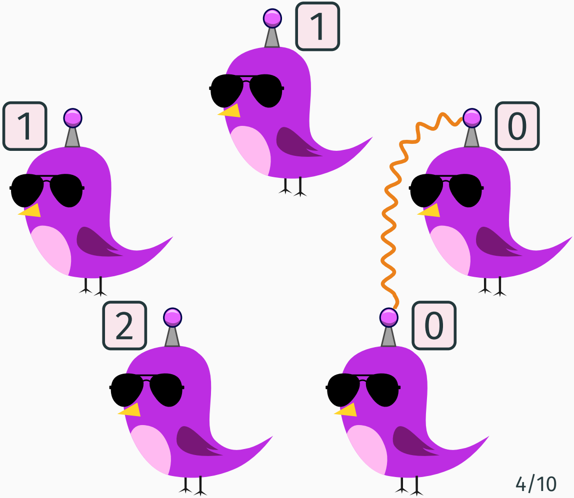

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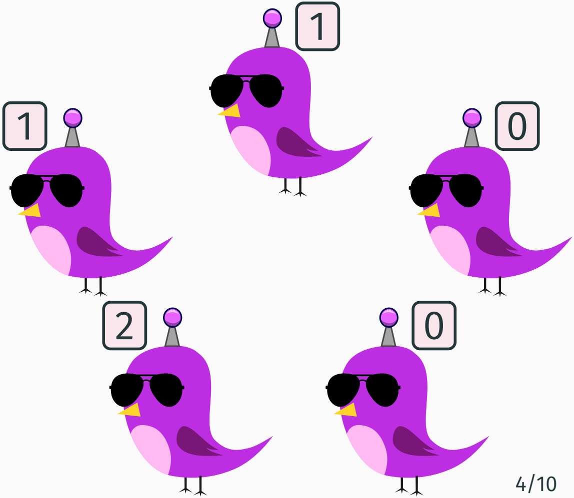

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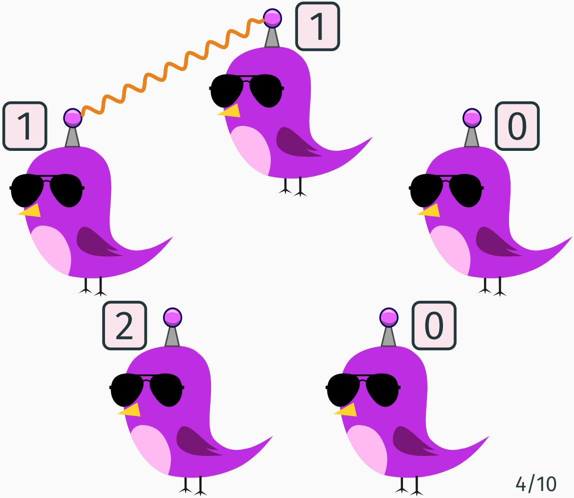

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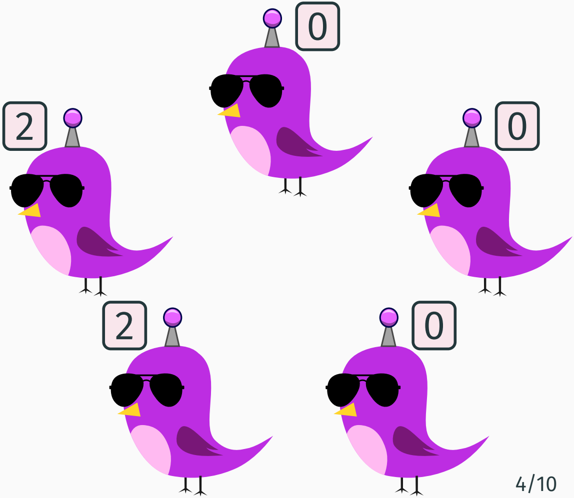

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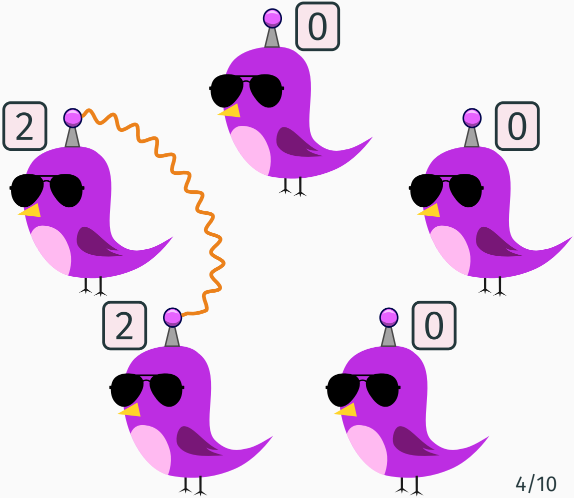

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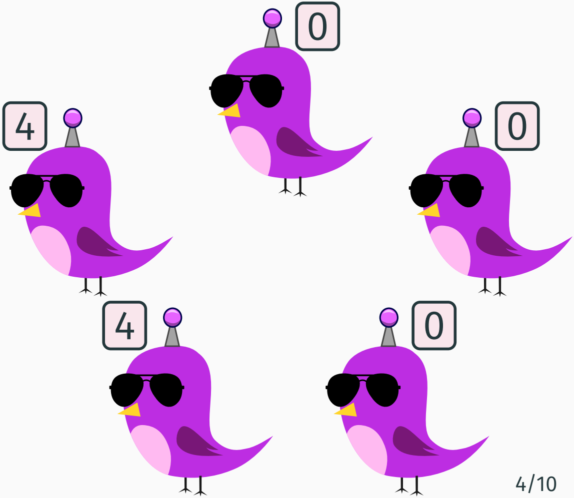

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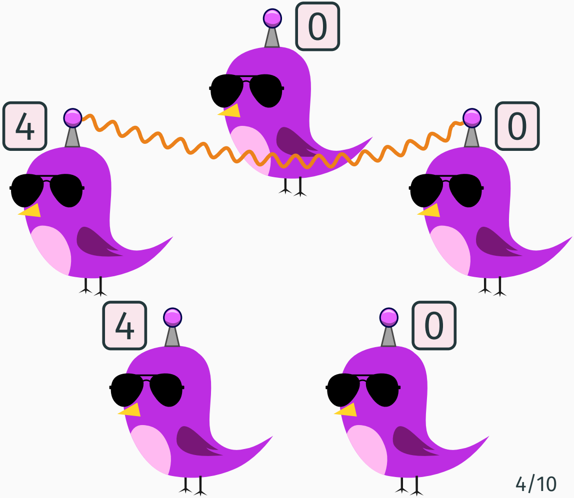

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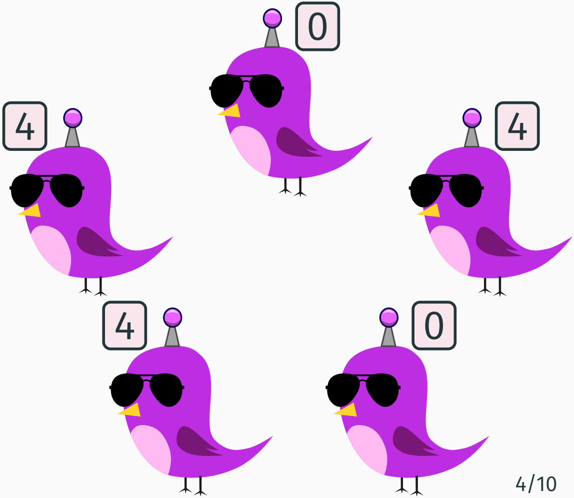

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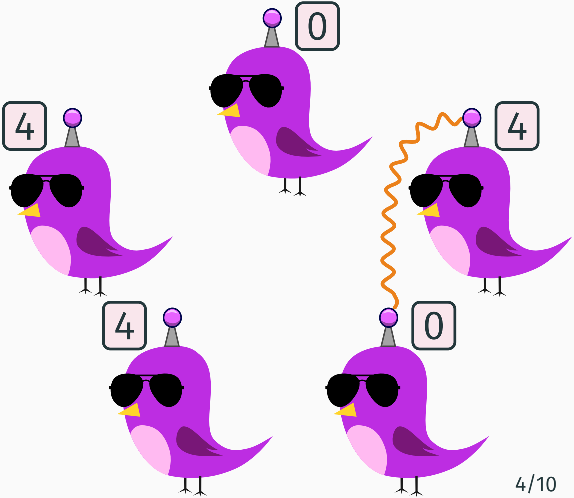

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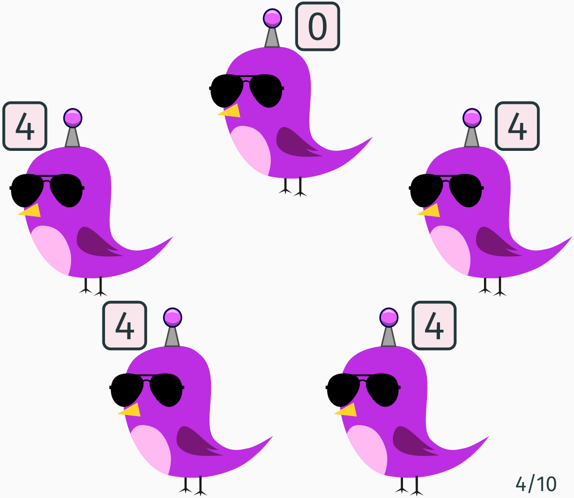

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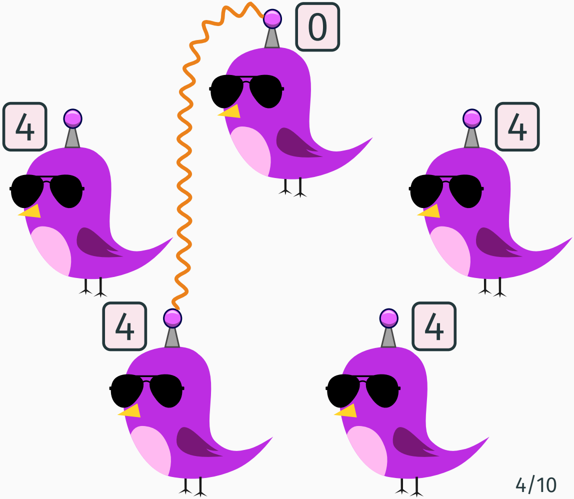

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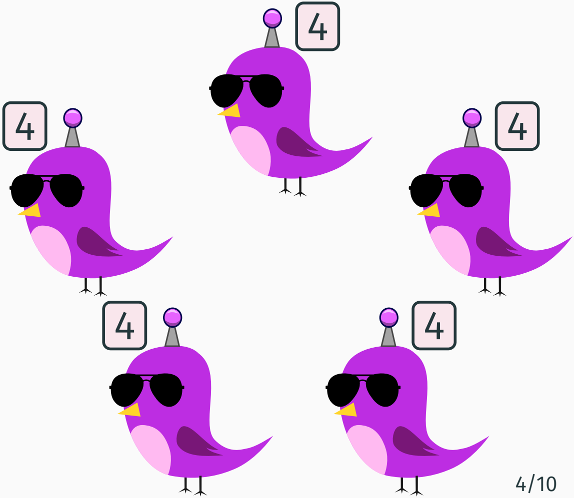

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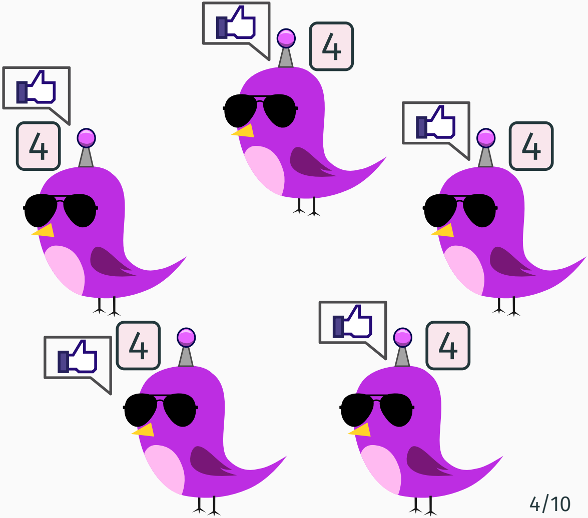
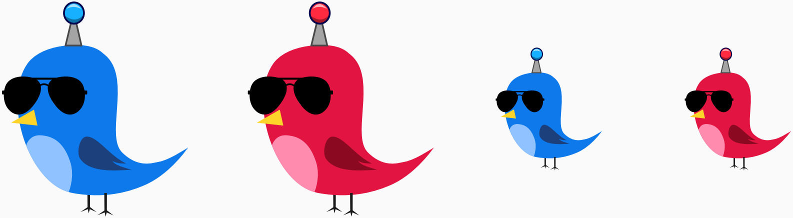


Demonstration

Population protocols: formal model

- *States*: finite set Q
- *Opinions*: $O : Q \to \{0, 1\}$
- *Initial states*: $I \subseteq Q$
- *Transitions*: $T \subseteq Q^2 \times Q^2$



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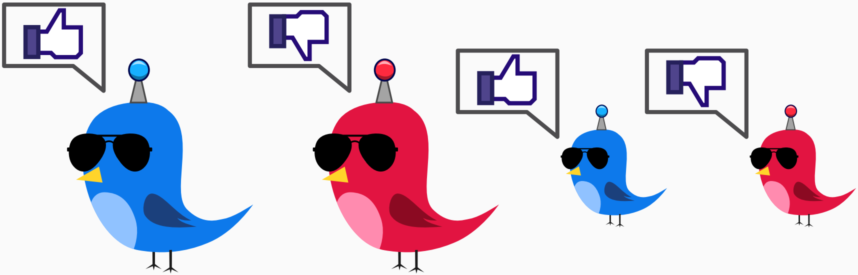

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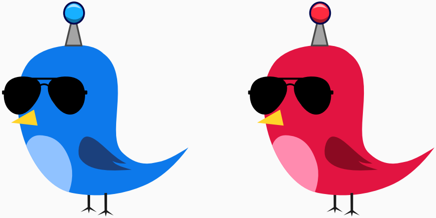

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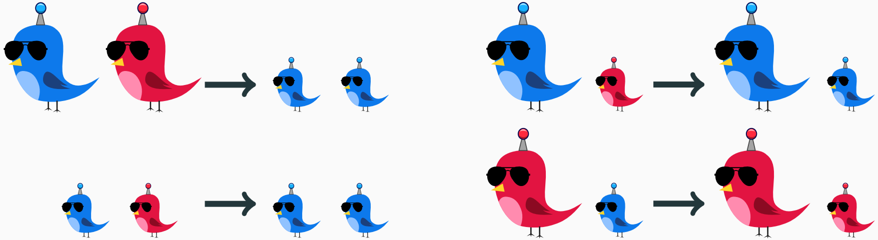
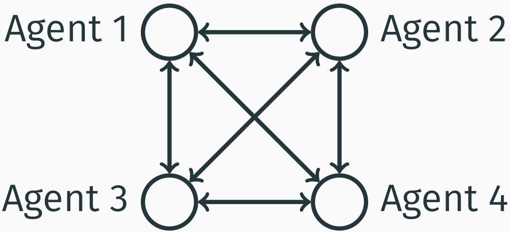
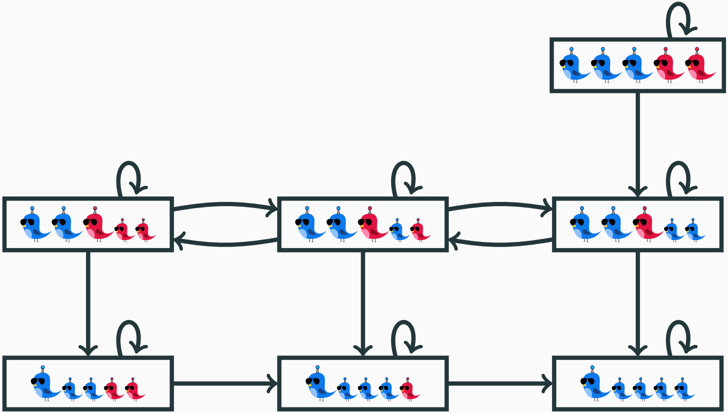


Interaction graph:

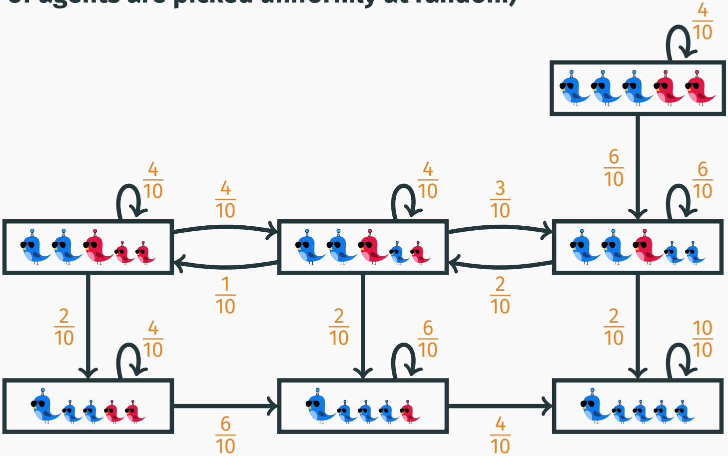


Reachability graph:

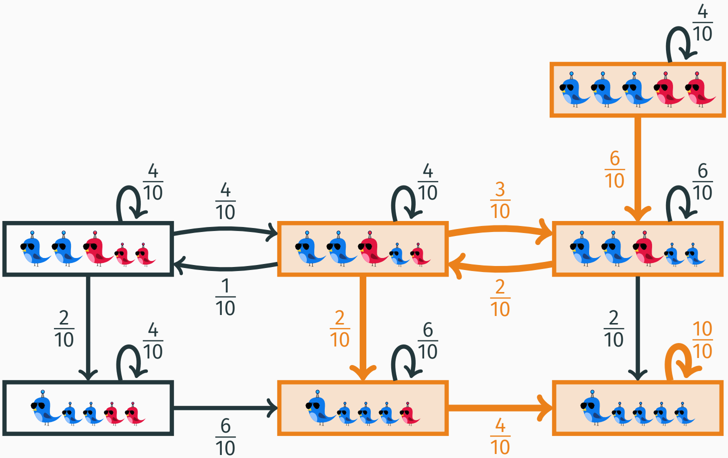


Underlying Markov chain:

(pairs of agents are picked uniformly at random)



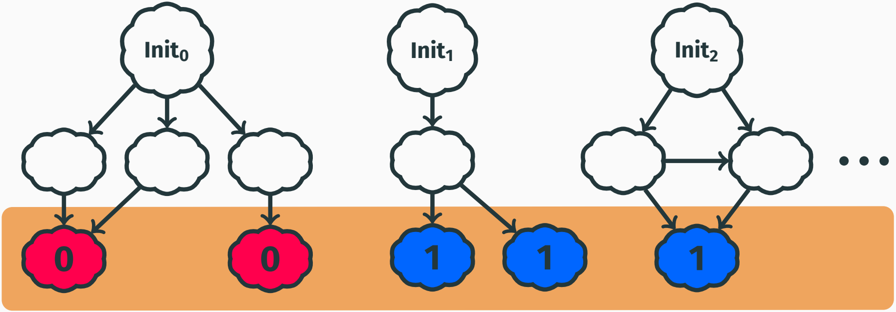
A *run* is an infinite path:



A protocol computes a predicate $\varphi\colon \mathbb{N}^I \to \{0,1\}$

if runs reach **common stable consensus**

with probability 1



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Expressive power **Angluin, Aspnes, Eisenstat PODC'06**

Population protocols compute precisely predicates definable in Presburger arithmetic, *i.e.* FO($\mathbb{N}$, $+$, $<$)

Other variants considered:

- Approximate protocols — *e.g.* Angluin, Aspnes, Eisenstat DISC'07
- Protocols with leaders — Angluin, Aspnes, Eisenstat Dist. Comput.'08
- Protocols with failures — Delporte-Gallet *et al.* DCOSS'06
- Trustful protocols — Bournez, Lefevre, Rabie DISC'13
- Mediated protocols, etc. — Michail, Chatzigiannakis, Spirakis TCS'11

Expressive power **Angluin, Aspnes, Eisenstat PODC'06**

Population protocols compute precisely predicates definable in Presburger arithmetic, *i.e.* $\mathrm{FO}(\mathbb{N}, +, <)$

Formal analysis of protocols

Protocols can become complex, even for B ≥ R:

Fast and Exact Majority in Population Protocols

Dan Alistarh
Microsoft Research

Rati Gelashvili*
MIT

Milan Vojnović
Microsoft Research

$$weight(x) = \begin{cases} |x| & \text{if } x \in StrongStates \text{ or } x \in WeakStates; \\ 1 & \text{if } x \in IntermediateStates. \end{cases}$$

$$sgn(x) = \begin{cases} 1 & \text{if } x \in \{+0, 1_d, \ldots, 1_1, 3, 5, \ldots, m\}; \\ -1 & \text{otherwise.} \end{cases}$$

$value(x) = sgn(x) \cdot weight(x)$

`/* Functions for rounding state interactions */`

$\phi(x) = -1_1$ if $x = -1$; 1_1 if $x = 1$; x, otherwise

$R_\downarrow(k) = \phi(k$ if k odd integer, $k - 1$ if k even$)$

$R_\uparrow(k) = \phi(k$ if k odd integer, $k + 1$ if k even$)$

$$Shift\text{-}to\text{-}Zero(x) = \begin{cases} -1_{j+1} & \text{if } x = -1_j \text{ for some index } j < d \\ 1_{j+1} & \text{if } x = 1_j \text{ for some index } j < d \\ x & \text{otherwise.} \end{cases}$$

$$Sign\text{-}to\text{-}Zero(x) = \begin{cases} +0 & \text{if } sgn(x) > 0 \\ -0 & \text{oherwise.} \end{cases}$$

procedure update$\langle x, y \rangle$
    if $(weight(x) > 0$ *and* $weight(y) > 1)$ ***or*** $(weight(y) > 0$ *and* $weight(x) > 1)$ **then**
        $x' \leftarrow R_\downarrow\left(\frac{value(x)+value(y)}{2}\right)$ and $y' \leftarrow R_\uparrow\left(\frac{value(x)+value(y)}{2}\right)$
    else if $weight(x) \cdot weight(y) = 0$ *and* $value(x) + value(y) > 0$ **then**
        if $weight(x) \neq 0$ **then** $x' \leftarrow Shift\text{-}to\text{-}Zero(x)$ and $y' \leftarrow Sign\text{-}to\text{-}Zero(x)$
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    else if $(x \in \{-1_d, +1_d\}$ *and* $weight(y) = 1$ *and* $sgn(x) \neq sgn(y))$ ***or***
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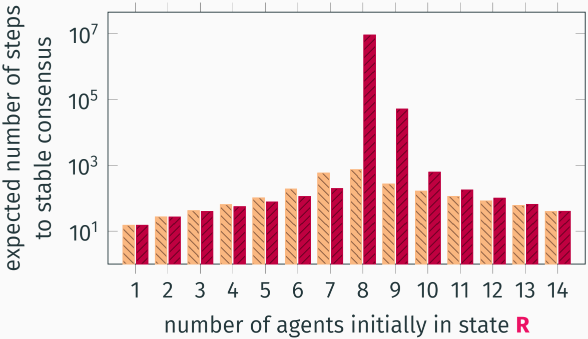
How to verify correctness automatically?

Convergence speed may vary wildly, challenging to establish bounds



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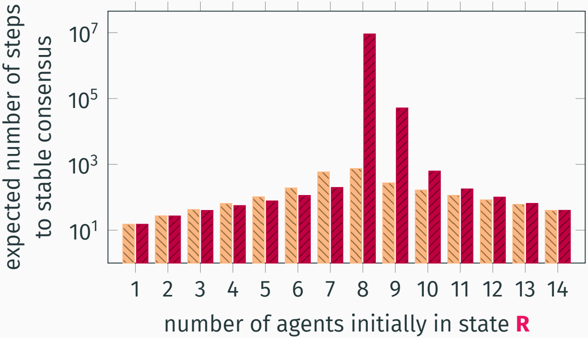

How to derive asymptotic bounds automatically?

Formal analysis of protocols

Number of states corresponds to amount of memory, relevant to keep it minimal for embedded systems

- **B ≥ R** requires at least 4 states (Mertzios *et al.* ICALP'14)
- **X ≥ c** requires at most c + 1 states

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What is the state complexity of common predicates?

Formal analysis of protocols

1. **Automatic verification of correctness**
   - Decidability — Esparza, Ganty, Leroux, Majumdar CONCUR'15, FSTTCS'16
   - Towards efficient verification — B., Esparza, Jaax, Meyer PODC'17
   - Complete tool — B., Esparza, Jaax CAV'18
2. **Automatic analysis of convergence speed**
   - First procedure — B., Esparza, Kučera (submitted to CONCUR'18)
3. **State complexity of protocols w.r.t. predicates**
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Existing verification tools:

- PAT: model checker with global fairness

  (Sun, Liu, Song Dong and Pang CAV'09)

- `bp-ver`: graph exploration

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- Conversion to counter machines + `PRISM/Spin`

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Verification: state of the art

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Only for populations of fixed size!

Sometimes possible to verify all sizes:

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Not automatic!

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Challenge: verifying automatically <u>all</u> sizes

Testing whether a protocol computes φ amounts to testing:

$$\neg\exists C, D: \quad \begin{aligned} & C \xrightarrow{*} D \wedge \\ & C \text{ is initial} \wedge \\ & D \text{ is bottom} \quad \wedge \\ & \text{opinion}(D) \neq \varphi(C) \end{aligned}$$

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As difficult as verification

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Relaxed with Presburger-definable overapproximation

Testing whether a protocol computes φ amounts to testing:

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$$D \text{ is bottom } \wedge$$
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Difficult to express

Testing whether a protocol computes φ amounts to testing:

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Most protocols are terminating!

Testing whether a protocol computes φ amounts to testing:

$$
\begin{aligned}
\neg\exists C, D: \quad & C \xrightarrow{*} D \;\land \\
& C \text{ is initial} \;\land \\
& D \text{ is terminal} \;\land \\
& \text{opinion}(D) \neq \varphi(C)
\end{aligned}
$$

Testable with an SMT solver

Testing whether a protocol computes φ amounts to testing:

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Protocol termination tested by structural analysis + SMT solving

Random variable *Steps*:

assigns to each run σ the smallest k s.t. σ_k in stable consensus

Maximal expected termination time

We are interested in $\mathit{time}\colon \mathbb{N} \to \mathbb{N}$ where

$$\mathit{time}(n) = \max\{\mathbb{E}_C[\mathit{Steps}] : C \text{ is initial and } |C| = n\}$$

Our approach:

- Most protocols are naturally designed in stages
- Construct these stages automatically
- Derive upper bounds on $time(n)$
  from stages structure

$\mathbf{B}, \mathbf{R} \mapsto \mathbf{b}, \mathbf{b}$

$\mathbf{B}, \mathbf{r} \mapsto \mathbf{B}, \mathbf{b}$

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$$(\mathbf{B} \vee \mathbf{R}) \wedge \bigwedge_{q \notin \{\mathbf{B},\mathbf{R}\}} \neg q \xrightarrow{\mathcal{O}(1)} \Box\left(\mathbf{B} \wedge \bigwedge_{q \neq \mathbf{B}} \neg q\right)$$

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$$(\mathbf{B} \vee \mathbf{R}) \wedge \bigwedge_{q \notin \{\mathbf{B},\mathbf{R}\}} \neg q \xrightarrow{\mathcal{O}(n^2 \log n)} \Box(\neg\mathbf{B} \vee \neg\mathbf{R}) \wedge \mathbf{b} \wedge \neg\mathbf{b}!$$

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$$\Box(\neg\mathbf{B} \vee \neg\mathbf{R}) \wedge \mathbf{b} \wedge \neg\mathbf{b}! \xrightarrow{\mathcal{O}(\exp(n))} \Box(\neg\mathbf{B} \wedge \mathbf{R} \wedge \neg\mathbf{b} \wedge \mathbf{r})$$

- Prototype implemented in python™ + Microsoft Z3
- Can report: $\mathcal{O}(1), \mathcal{O}(n^2), \mathcal{O}(n^2 \log n), \mathcal{O}(n^3), \mathcal{O}(\text{poly}(n))$ or $\mathcal{O}(\exp(n))$
- Tested on various protocols from the literature

Peregrine: Haskell + Microsoft Z3 + JavaScript

`peregrine.model.in.tum.de`

- Design of protocols
- Manual and automatic simulation
- Statistics of properties such as termination time
- Automatic verification of correctness
- More to come!

Demonstration

Population protocols can be formally analyzed automatically:

- Verification of correctness
- Analysis of expected termination time
- Tool support!

Ongoing investigation of state complexity

Conclusion: future work (seeking for PhD students/Postdocs)

ERC Advanced Grant —

PaVeS: Parameterized Verification and Synthesis

- Goal: Develop proof and synthesis techniques for distributed algorithms working correctly for an arbitrary number of processes
- PI: Javier Esparza (esparza@in.tum.de), TU Munich
- Start of the project: Sept. 1, 2018
- Start of the PhDs/Postdocs: flexible, from Sept. 1, 2018 to about Sept. 1, 2019

Thank you!

Vielen Dank!